Binary scattering as an analytical tool in TLM heat-flow models

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Abstract. In this paper we develop methods for analysing the accuracy of dynamically resized meshes in TLM thermal models. However, the binary scattering approach which is used has wider applications and we demonstrate how it can lead to a greater understanding of the underlying processes in TLM. We also demonstrate how it can be used to provide a novel error parameter.

Introduction. There are instances in heat-flow modelling where the point/s of excitation are small compared with the overall problem space under consideration. So, instead of doing the TLM process of scatter connect on a large number of zeros, it shoud be possible to truncate the mesh and dynamically expand it on front of the expanding thermal profile. This concept is not new [1,2], but until now the question of how precisely this mesh should expand has not been addressed. A single shot excitation of a TLM mesh which is allowed to propagate during k iterations leads to a Gaussian profile. A Gaussian profile has an associated standard deviation σ and statistics indicates the extent of error if the tail beyond nσ (n = 1,2,3, etc) is truncated. In this paper we demonstrate how the concept of binary scattering can be used in the statistical analysis of this problem. We also demonstrate how it can be used in the formulation of a new error parameter. The truncated mesh is applied to the problem of heat-flow in complex geometries.

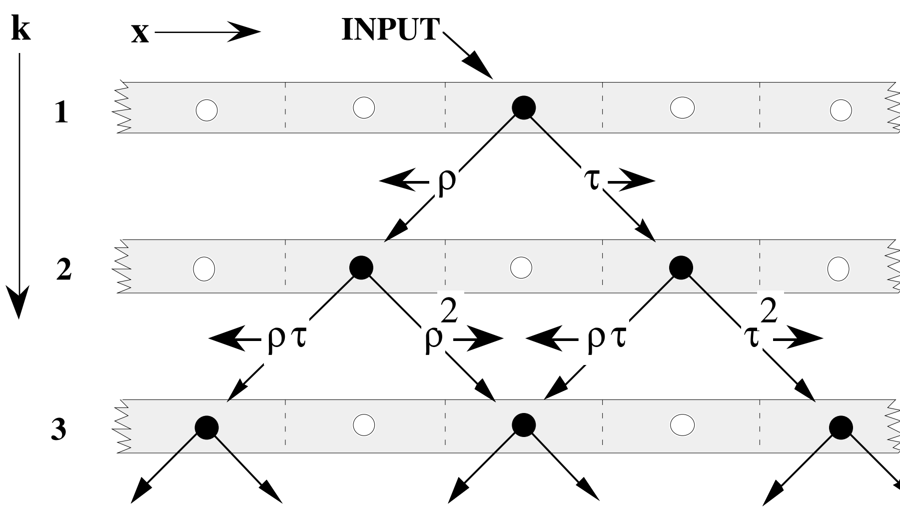

Figure 1 One-dimensional binary scattering as a function of position and time following single-shot excitation.

Binary scattering.
The concentration at any position (x) after k iterations is then the superposition of all scatter contributions from within a 'region of dependence'. Enders & de Cogan [3] have commented on the problems of calculating this concentration in an analytical way. Although the number of contributions at any point can be described by a binomial coefficient, it must be remembered that each point is a generator for a new Pascal triangle. The problem was addressed combinatorially by Moravec [4] who provided expressions for uni-directional injection (as here) and the more realistic injection to left and right.

If the scatter contributions at iteration (k) are arranged in power order at each position then this separation provides an interesting insight. Table 1 provides an example corresponding to k = 5.

Table 1

components by exponent		(x-5)	(x-3)	(x-1)	(x+1)	(x+3)	(x+5)	x-positions occupied
1	τ^5						1	1
5	$\rho\tau^4$	1	1	1	1	1		5
10	$\rho^2\tau^3$		1	2	3	4		4
10	$\rho^3\tau^2$		3	4	3			3
5	$\rho^4\tau$			2	3			2
1	ρ^5			1				1
components/position		1	5	10	10	5	1	

We see that the highest power in τ (τ^k) is furthest to the right. The next power down ($\rho\tau^{k-1}$) has one contribution at every position except (x+k). The lower powers are distributed according to rules which depend on whether k is odd or even. We can now insert actual values for ρ and τ. If ρ is very small then contributions containing powers of ρ^2 and higher can be neglected. The profile then comprises a pulse of magnitude τ^k at position (x+k) with a tail of constant magnitude $\rho\tau^{k-1}$ which stretches from (x-k) to (x+k-1), exactly what would be obtained in the solution of a lossy hyperbolic equation. If the value of ρ is such that higher terms cannot be neglected then the resulting profile moves from hyperbolic to parabolic behaviour. Recent work on subdivision of lossy TLM nodes has come to a similar conclusion [5]. This has demonstrated that if a constant velocity is maintained, then as the space and time discretisations in a TLM diffusion model are reduced the result is equivalent to the summation of many solutions of the telegraphers' equation.

A new error parameter: Since the inception of TLM heat-flow modelling a variety of error parameters have been developed to determine the wave content within a propagating wave-form. This is important since there has been a need to justify why the lossy wave equation of TLM has been used to model diffusion processes. This process becomes trivially easy if we use the information in the previous section. We can estimate the wavelike contribution either globally or at each node. The global estimation is the ratio of the hyperbolic contributions to all contributions.

$$\frac{\tau^k + k\rho\tau^{k-1}}{\tau^k + k\rho\tau^{k-1} + \frac{k^2}{2!}\rho^2\tau^{k-2} + \frac{k^3}{3!}\rho^3\tau^{k-3}. \quad . \quad . \quad . + \rho^k} \tag{1}$$

A plot of eqn (1) for different values of ρ and k provides a useful error surface.

Statistics of binary scattering: Numerical experiments with one-dimensional single-shot injection using TLM have indicated a relationship between the variance, σ^2, the time-step, k and the scattering parameters ρ and τ [6]. For example, the variance for link-line TLM configurations was found to follow the relationship:

$$\sigma^2(k) = \frac{\tau}{\rho}k + \frac{1}{2} - \frac{\tau}{\rho}\left[1 - \frac{\tau}{2\rho}\right] \tag{2}$$

The analytical solution for diffusion from a limited source yields a variance $\sigma^2 = Dt$ (D is the diffusion constant and t is the time). If we use discretised space and time then we can show that

$$D = \frac{\Delta x^2}{\Delta t}\frac{\tau}{\rho} \tag{3}$$

Since Dt = D kΔt, It follows that $\sigma^2 = \Delta x^2 \frac{\tau}{\rho} k$ (4)

It could be inferred that differences between (2) and (4) are due to errors in the TLM description of diffusion phenomena, although they could also be due to the way in which injection is implemented in these models. Ongoing work in this investigation involves using the Moravec method to derive the contributions in a calculation of variance.

Application of truncated meshes in heat-flow

Lee [1] identified the enormous savings that could be achieved by expanding a mesh dynamically in advance of a thermal transient. There was no consideration of accuracy; the algorithm merely kept the mesh 4Δx in front of the extreme node with a 1% rise above ambient. In a study of heat-flow into electric motor cooling fins after switch-on we have estimated the effects of expanding a mesh as nσ(k) (n = 1, 2, 3 . . .). Tests on single shot excitation in a one-dimensional mesh indicate that dynamic expansion as σ(k) gives significant errors, but expansion as 3σ(k) gives very reasonable results over most of the time of the run. The transition to a two-dimensional representation of a fin with single shot excitation suggests that 4σ(k) might be even better. However, the results shown in figure 2 confirm that 3σ(k) expansion is satisfactory for constant temperature input along the lower surface of the fin model.

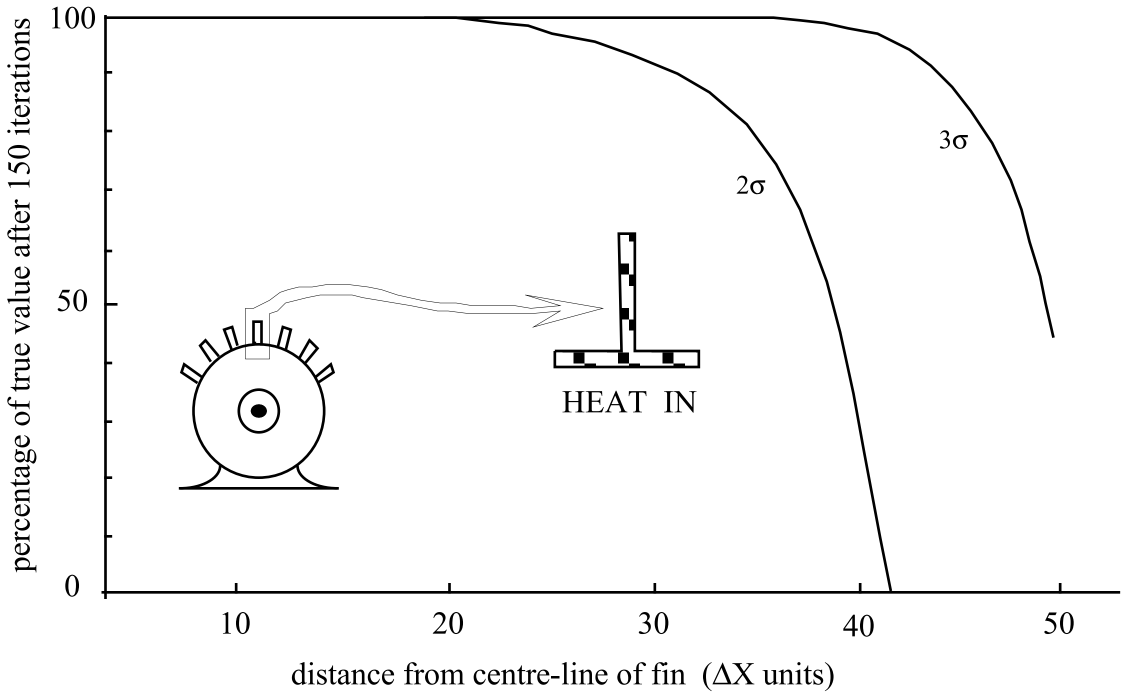

Figure 2 Accuracy (defined as $100*T_{truncated}(x)/T_{true}(x)$) as a function of distance from the centre-line of the motor cooling fin after 150 iterations for $2\sigma(k)$ and $3\sigma(k)$ mesh expansions.

Conclusions

This work has demonstrated that binary scattering is a very useful tool in a range of analyses associated with heat-flow modelling using TLM. There is still much work to be done in one dimension and sometime soon someone is going to have to face the daunting task of extending the technique to two and three dimensions.

References

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