A TLM model of a heavy gantry crane system

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Abstract

The 1-D wave equation model of a vibrating string is first adjusted to allow for variable tension. The response is verified by comparison with a known analytical solution for vibrations in a rotating cable, where centrifugal forces cause tension variation with length. Then the vibrations of a dangling cable are modelled where the weight of the cable under gravity adds an extra restoring force to the normal cable tension component. Again the result is verified by comparison with a known analytical solution. Finally, the two ideas are combined to model the motion of a gantry crane, where the motion of a trolley on a gantry rail is used to position a hanging load mass via a heavy cable. The full system can now undergo a net displacement rather than vibration about a neutral position. The model successfully simulates the entire dynamics of manoeuvring the hanging load including cable gravity, variable tension and variable cable density effects.

1. Introduction

The 1-D wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

bears this name even though the majority of waves are not governed by it! [1]. The equation arises primarily in electromagnetics and mechanics, especially acoustics. Its derivation involves simplifying assumptions the validity of which depend strongly on the problem. TLM is a well established, time-domain numerical technique for solving the wave equation. Recently, considerable work has focussed on refining the basic TLM algorithm to account for significant departures from "ideal" wave behaviour when one or more of the simplifying assumptions no longer apply [2-4].

Perhaps the most commonly derived wave equation in physics is that of the vibrating string under tension. The derivation assumes that the string is perfectly flexible and uniform, that gravity effects are negligible, that the tension is constant along the string, that the amplitude of vibration is small, and that damping is negligible. Under these assumptions, standard 1-D TLM will elegantly solve the vibrating string problem.

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These assumptions lose their validity for example in a heavy cable in a gantry crane system, such as in Fig.3. The cable carries a load mass at the lower end and is attached to a trolley at the top. The tension in the cable will vary with height due to the cable's own weight, the swings can be large, gravity adds a new restoring force to the normal component of tension, and the cable may not be uniform. Each of these effects causes a departure from Eqn (1).

The general TLM model for this case is here developed in three stages. Firstly, a model is developed for the small-amplitude vibrations of a light, homogenous string, fixed at one end and rotating freely about this fixed end. The novelty here is that the tension varies along the length of the cable. Then a TLM model of a hanging cable under gravity is considered, in which gravity adds an external restoring force when the cable locally departs from the neutral, vertical position. These cases are chosen because analytical solutions are available for both, allowing verification of the TLM model. Finally, the TLM model of the full gantry crane is presented, involving the additional novelty of net translations of the entire system.

2.1 Rotating string: differential equation and analytical solution

For the light, rotating string, if gravity and air-resistance are neglected, the "equilibrium" position of the string will be a straight line rotating with angular velocity ω in a plane passing through the fixed point of rotation. It is assumed that the amplitude of vibrations is small, and that the string displacement from the equilibrium is parallel to the axis of rotation and perpendicular to the plane of rotation. Associated with the rotation is a tension supplying the centripetal acceleration of the string mass from any point to the end of the string. This varies along the string, from a maximum at the centre, to a value of zero if the end is free, or to $ml\omega^2$ if the string, length l, is terminated by a lumped mass m.

The differential equation describing the displacement $u(x,t)$ of any point on the string from the "equilibrium" rotating straight line is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial}{\partial x}\left[\left(l^2 - x^2\right)\frac{\partial u}{\partial x}\right] \tag{2}$$

where $c^2=\omega^2/2$ and x is the distance from the fixed point [5]. The general solution is expressible as

$$u(x,t) = \sum \left\{A_m \cos[\sqrt{2m(2m-1)}ct] + B_m \sin[\sqrt{2m(m-1)}ct]\right\} \quad P_{2m-1}(x/l) \tag{3}$$

where P_n is an nth order Legendre polynomial, m = 1,2, … , and the constants A_m and B_m are determined by the initial conditions.

2.2 Rotating string: TLM model

The varying tension in the string causes a continuously varying wave speed and wave impedance along the string. This can be modelled in the TLM scheme by inductive stubs of varying inductance (impedance). As the inductance is inversely proportional to the tension, and stubs increase the line's inductance, the region of highest tension (fixed point) will have no stubs, with the stub inductance growing towards the regions of lowest tension (end point). Under the assumption of low amplitude vibration, the tension distribution and therefore the stub inductances can be assumed to be unvarying with time.

2.3 Rotating string: results

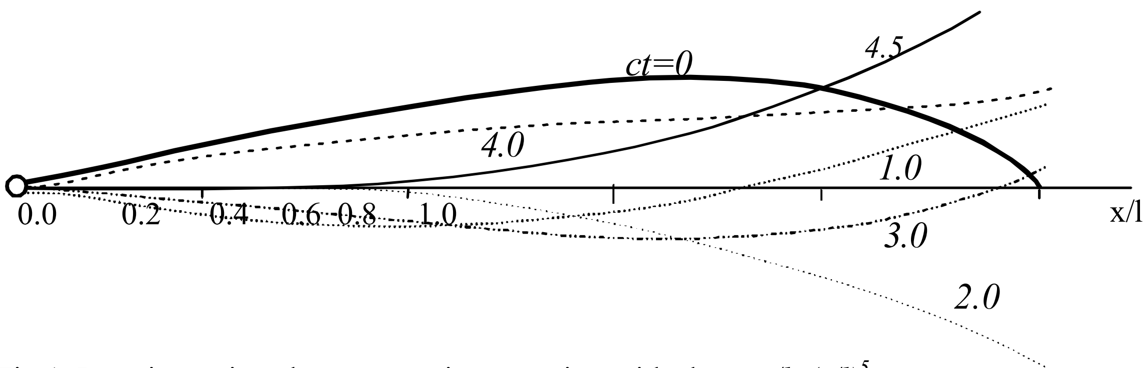

Fig.1. Rotating string shapes over time, starting with shape $x/l-(x/l)^5$.

Figure 1 shows a summary superposition of results. An initial shape $u(x,0) = x/l-(x/l)^5$ is assumed ($ct=0$), with zero initial velocity $u_t(x,0)=0$, and zero end mass. Snapshots of the waveform are shown for successive values of ct from 0 to 4.5. The analytical and TLM solutions agree well provided the variation in tension is within reasonable limits.

3.1 The hanging cable: introduction

Like the rotating string the hanging cable also has a varying tension along the length. But it has another departure from the wave equation in that gravity supplies a second, deflection-related force (in addition to the normal component of the tension force) tending to restore the cable to its neutral position.

Regarding boundary conditions, as with the rotating string, at one end (the top) there is simply a fixed point. At the lower end there are at least three boundary conditions of practical interest: a free end, a fixed end, or an inertial, hanging load free to vibrate (or "swing") laterally. In the third case the cable vibrations "tug" at the load mass, causing it to accelerate. The moving load, in turn, then drives the cable motion. If of interest, this coupling of the two dynamic systems must also be modelled in TLM.

3.2 Hanging cable: Analytical analysis and results

The differential equation governing small amplitude vibration of a vertical flexible cable under gravity is

$$\frac{\partial^2 u}{\partial t^2} = g\left(x\frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial t}\right) \tag{4}$$

where u is the lateral, horizontal displacement of the cable, and x is now the distance from the *lower* end. By assuming synchronous motion and therefore a solution separable into time and space components, $u = U(x)\cos\omega t$, the space part $U(x)$, for the free end case, can be shown to obey a Bessel's differential equation [6],

$$\frac{d^2U(z)}{dz^2} + \frac{1}{z}\frac{dU(z)}{dz} + U(z) = 0 \tag{5}$$

where $z=2\omega\sqrt{(x/g)}$. The solution for one particular frequency (mode of vibration) is $U(z) = J_0(z)$ or $U(x) = J_0(2\omega\sqrt{x/g})$. Again, the general solution can be made up as an infinite sum of such terms for all frequencies and modes.

3.3 Hanging cable: TLM model

For the basic TLM variable in this case, there are significant advantages in choosing, not the cable lateral displacement, u, but the variable $-T(\partial u/\partial x)$ corresponding to the normal force component of the tension. The negative sign is because a negative gradient in a wave travelling in the positive x-direction produces a positive force. This formulation greatly facilitates the force perturbation approach described below. The lateral velocity, $\partial u/\partial t$, at any point in the cable is then given by $(p-q)/Z$, where $Z=\sqrt{(\rho T)}$, is the mechanical impedance of the cable, with ρ the mass per unit length. The instantaneous position of any point on the cable can be obtained by time integration of the velocity, or by spatial integration of the slope.

The extra restoring force at any point on the cable is $-\rho.\Delta l.g.\partial u/\partial x$, which is the normal component of the gravity force shown in Fig.2. This force constitutes a perturbation on the wave, the effects of which can be shown to cause two equal variations to the force waves, that travel in opposite directions. So half of this gravity force should be added directly to each of the two counter-propagating TLM pulses at every point on the cable.

The cable tension at the lower end is the load weight (if present) or zero (if free). The tension rises with distance up the cable, reaching the total weight of the cable+load at the upper, fixed end. As before, this is modelled by adding inductive stubs of varying impedance.

The coupling of the moving boundary at a load mass and the cable TLM is as follows: The lateral acceleration of the load is the total lateral tension force, $-T\partial u/\partial x$, at the base of the cable, divided by the mass. This is integrated over time to get the instantaneous velocity, *v*, of the load. The reflection boundary condition for the lower end of the cable is modified by the addition of an amount *vZ* to the reflected pulse at each time step.

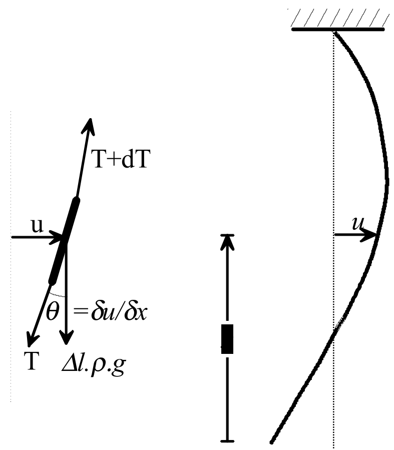

Fig.2. The forces (left) on an string element of a hanging cable under gravity, and (right) the first mode of vibration of this system.

3.4Hanging cable: results

Again, TLM results are found to agree with known analytical results for simple cases. Figure 2 shows the first mode of vibration.

4.1 The gantry crane model

The system to be modelled is shown in Fig.3.

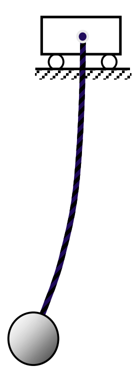

Fig.3. The gantry crane configuration, with trolley, cable and load.

The modelling of the variable tension due to the cable's own weight, and the effects of gravity on the cable vibration were modelled as described above. From a TLM modelling point of view, a third new issue arises. The system is not just vibrating (or "swinging") about a neutral axis, but the entire system can undergo a net movement over an arbitrary distance as the trolley travels. Two ways of dealing with this were investigated. At each time step, the new system configuration was obtained by temporal integration of the velocity of each cable element, $(p-q)/Z$, from the known initial position. A second approach was to obtain the cable and load configuration by spatial integration of the cable slope from the known trolley position. The two methods gave mutually consistent results.

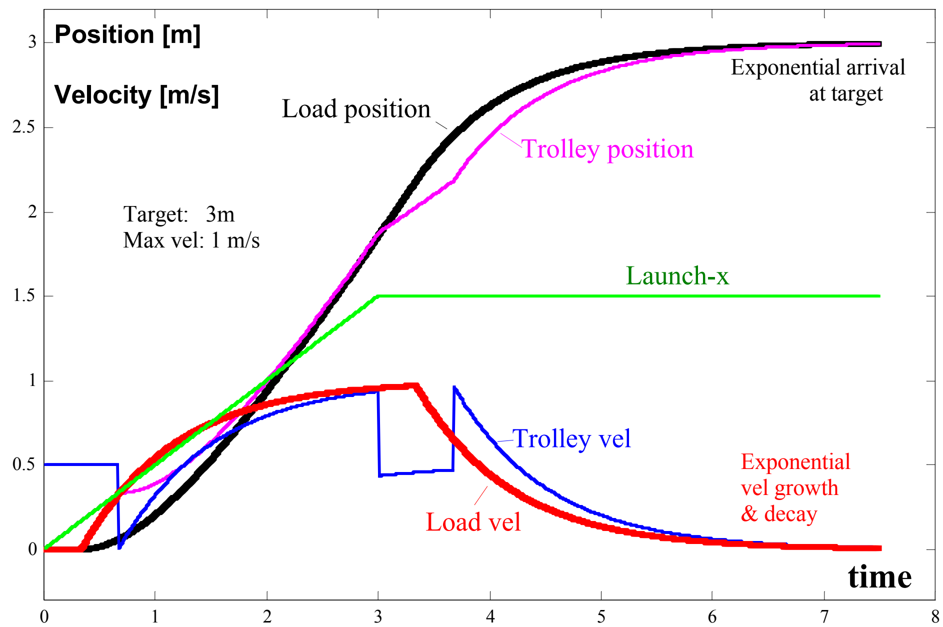

Fig.4 Simulation of a gantry crane manoeuvre, using a full TLM model.

Figure 4 illustrates an application of the model to test a novel control strategy (the topic of another paper) in which the aim is to move the load from rest to a target position by combining position control and active swing control of the load mass.

5 Conclusion and future work

Novel techniques have been presented for successfully modifying 1-D TLM to model important physical effects causing different kinds of departure from the wave equation for a vibrating string or cable. A combination of varying impedance stubs and "force perturbation" is used for the primary physical effects, while the net movement of the gantry crane is achieved by integration. Known analytical results for special cases were used to test the ideas. Once again, TLM has proven capable of adaptation to model successfully important physical effects and the range of its application areas continues to expand.

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