Matlab representations of Polar Transmission Line Matrix Meshes.

D.S.Hindmarsh[+]

Abstract

This paper discusses some of the issues that evolve when representing Polar Transmission Line Matrix models using normal TLM techniques with the Matlab software package.

Introduction

TLM is a modelling tool developed using a time domain algorithm, this can be used to solve electromagnetic and many other wave flow problems. An issue with the normal representation of TLM is that it takes the form of a mesh, made from inter crossing nodal positioning. This allows for normal two dimensional representations through flat plains. However due to the nature of the rectangular uniformity of the mesh when applying boundaries that are not in line with the linearity of the nodes, we have to apply different techniques to the boundary to perform curved, radial, and angled boundaries. The problern is further compounded when we venture in to three dimensions, and produce spherical boundaries, and or cylindrical pipe networks. For this we need to embark into the Polar method of TLM Modelling.

One of the programming languages used commonly for TLM modelling is Matlab. Maltlab has the advantages of ease of user interface and good visual representational capabilities. To date the author has produced many models in two dimensional flat space for acoustical problems. These problems have often required reflecting boundaries that have not been a straight line of nodes with their scatter and connect process changed. When curves are required we have to adopt the `Step-wise' approach, which, though gives a rough curved representation, at lower resolutions when the number of nodes is reduced the accuracy of using such boundaries becomes impossible. Cacoveanu et al [1] have carried out research into deriving effective methods of applying the polar TLM method. Which proves to be very useful when considering pipe networks.

The theory of Polar Meshes in Electromagnetic terms applied to accoustic problems

There has been some work, a long time ago, on polar meshes for electromagnetic modelling where the resistance is zero and the capacitance is related only to nodal volume. It is believed that this is probably the first time that polar TLM meshes have been applied to acoustic problems

[+] School of Information Systems, University of East Anglia, Norwich NR4 7TJ

Polar Meshes for lossless TLM models with axial symmetry were first developed by Al-Mukhtar and Stich [2] and examined in detail by Naylor [3]. de Cogan and John [4] extended the technique to diffusion problems, with further work in orthogonal mesh types by Meliani [5].

The polar mesh has generally been developed for thermal diffusion modelling where each node in a cylindrical problem has capacitance given by the product of material density, volume and specific heat. The thermal resistance given by the length divided by the product of area and thermal conductivity. The volume may be calculated as the nodal area (defined by integration) times the nodal height (Δh). there is however a question when we come to calculate the area of radial flux. $\Delta h(\theta(r+\Delta r))$ gives an over-estimate, while $\Delta h(\theta r)$ is an under estimate. Most TLM modellers use the mean value.

The area corresponding to the radial centre is obviously zero and therefore the impeadance of the point must be infinite, $\rho = 1$ follows from an electromagnetic definition of reflection coefficient. The radial centre in a homogeneous problem is also a centre of symmetry. Thus, whatever flows from right to left through the point must be matched by an equal flow from left to right [6]. This assumes that if the propagation behaves symmetrically for a section of the circle we need only model that section and assume all sections to be the same. However, when this is not the case, e.g. the problem is asymmetric (e.g. certain vibration modes on a drum skin) then we cannot use this simplification and the entire 360° must be modelled.

Following the basic function of a polar mesh, we developed a polar representation of a simple waveform scatter, using a square mesh with the characteristics and function of a polar propagation. However this still does not visually represent a pipe section. The next task was to code together the flat functional representation of the polar propagation and form it into a circular plot, which would then give us proof of the scattering velocities, and show a pipe cross section.

One of the problems of using a rectangular mesh representation for polar meshes is that the mesh must obey ‘folding symmetry’. Whereby data must be passed between the end of the mesh where θ is equal to 360°, and 0° at the opposite end of the mesh along the length ‘r’ (radius) of the mesh. See below;

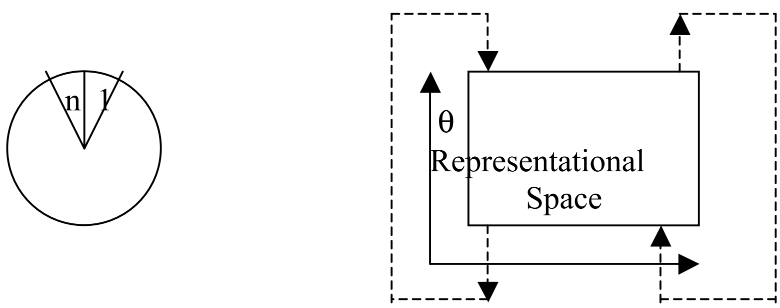

Figure 1. ‘Fold over’ Symmetry

From the work carried out by de Cogan [6] it can be put forward that the centre point or middle of the polar mesh when seen in a two dimensional plane, has to be considered as a circle of infinitesimal circumference [1]. This isr shown in figure 2.

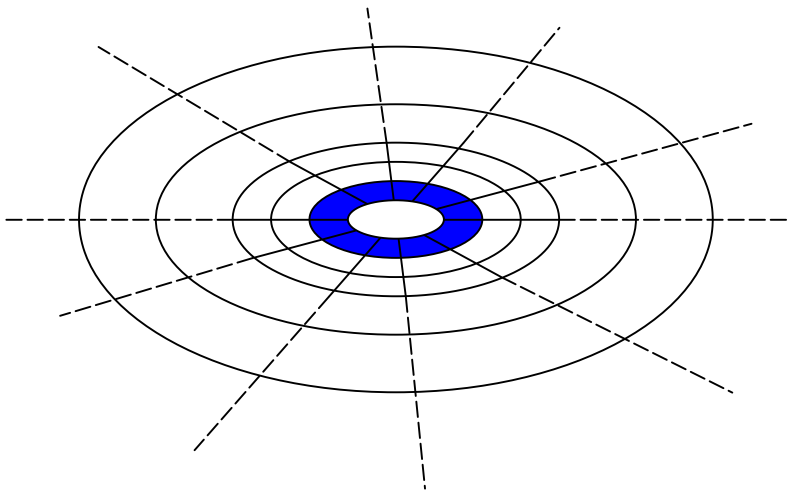

Figure 2 Close up of centre point within the polar mesh

Figure 2, shows that the centre circle of the polar mesh can be seen to be of the same dimensions as the square TLM mesh side, the edge has been bent round. Providing the scatter and connect across such an area is correct, (discussed later) this provides an accurate polar representation. By using the same basic technique for the TLM scatter and connect process and only applying it to a polar representation we have reduced the complexity of the TLM model.

Although polar TLM and Cartesian mesh TLM may seem similar in the first instance, there are some important considerations. The very nature of a polar mesh is that the nodes are not equi-distant from each other. This means that for every node moved from the centre of the polar mesh a different calculation has to be made to compensate for the change in the distance from each node as the circumference increases. The more usual technique that allows for changes in nodal values is the addition of `stub' lines.

TLM nodes with stubs.

Stubs have for a long time been used in microwave engineering. The concept can be applied to the study of polar mesh representation by allowing the normal, flat, TLM to have the effect of a polar mesh. The main feature of a stub is that it acts as storage rather than a `leakage' element. This means that data that is passed to it during the matching process is returned to the network and not lost from the network. If the stub is terminated in an open-circuit ($\rho = 1$) then the `phase' of the data will be retained. If it were a short-circuit ($\rho = -1$) then the `phase' of the data would be inverted. A stub line of length $\Delta x/2$ will return data to the network after one time step: a pulse will take a time Δt/2 to reach the termination and Δt/2 to return.

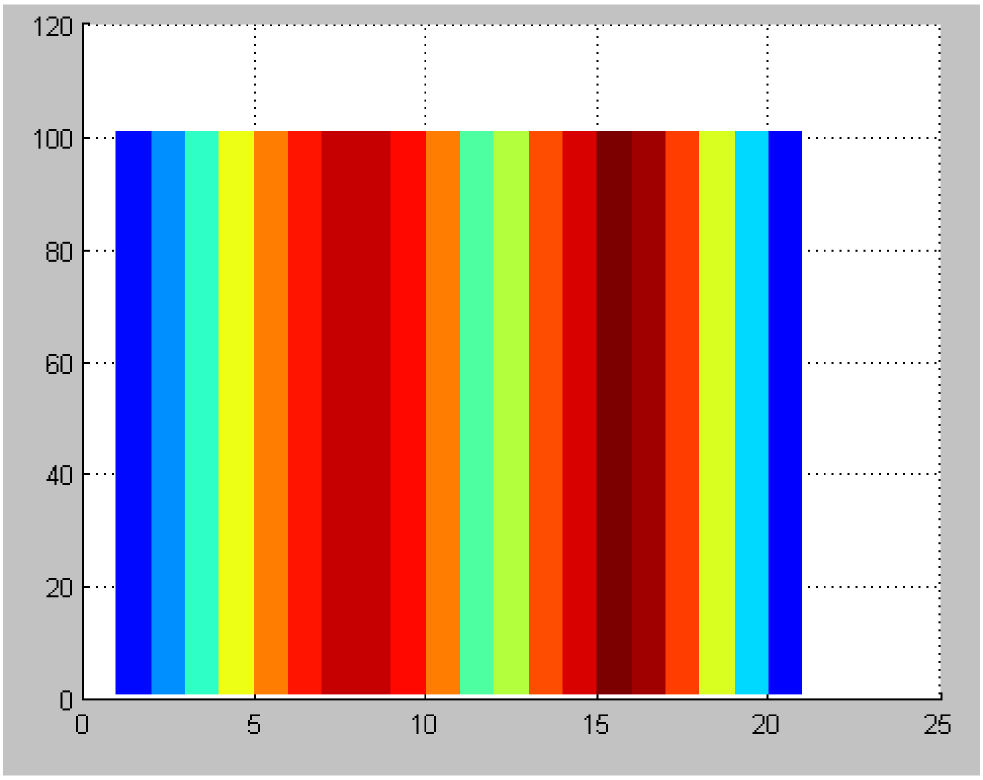

Figure 3 Response on a Cartesian mesh of an excited network which is stub-loaded in accordance with the geometrical differences in nodal volume in a polar problem

In TLM algorithms it is normal to use an open-circuit terminated stub to match changes in capacitance and short circuit terminated stubs to match changes in inductance [6]. The addition of the stubs does have an increase in the complexities of the TLM algorithm. The increase in computational effoty is relative to the increase in the TLM scatter matrix. The difference between a normal TLM scatter matrix and a scatter matrix with Stubs can be seen below.

no stub

$$\begin{pmatrix} {}^sV_N \\ {}^sV_S \\ {}^sV_E \\ {}^sV_W \end{pmatrix}_k = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}_k \begin{pmatrix} {}^iV_N \\ {}^iV_S \\ {}^iV_E \\ {}^iV_W \end{pmatrix} \tag{1}$$

with stub

$$\begin{pmatrix} {}^sV_N \\ {}^sV_S \\ {}^sV_E \\ {}^sV_W \\ {}^sV_{st} \end{pmatrix} = \frac{1}{4Z_s + Z} S \begin{pmatrix} {}^iV_N \\ {}^iV_S \\ {}^iV_E \\ {}^iV_W \\ {}^iV_{st} \end{pmatrix} \tag{2a}$$

where $$S=\begin{pmatrix} -(Z+2Z_s) & 2Z_s & 2Z_s & 2Z_s & 2Z \\ 2Z_s & -(Z+2Z_s) & 2Z_s & 2Z_s & 2Z \\ 2Z_s & 2Z_s & -(Z+2Z_s) & 2Z_s & 2Z \\ 2Z_s & 2Z_s & 2Z_s & -(Z+2Z_s) & 2Z \\ 2Z_s & 2Z_s & 2Z_s & 2Z_s & (Z-4Z_s) \end{pmatrix} \tag{2b}$$

Applying the stubs means that the equivalent of the polar mesh can be created in one flat piece (see figure 3). This section or plain can be stretched around a fixed point and joined together along leading and following edges (see figure 1). This technique, mentioned earlier, is a computationally faster process as it is only the physical representation, which the user sees that is being altered.

In figure 3, the square representation can be thought of as a skin which has all the characteristics of a polar representation, (not counting the centre-node problem which will be discussed later). This flat mesh has a stub distribution that changes as the dimensions and resolution change for each application. Therefore, for a large pipe section or surface area the value of each stub has to be recalculated to produce the correct scattering for a different arc and area size. This is of course user alterable.

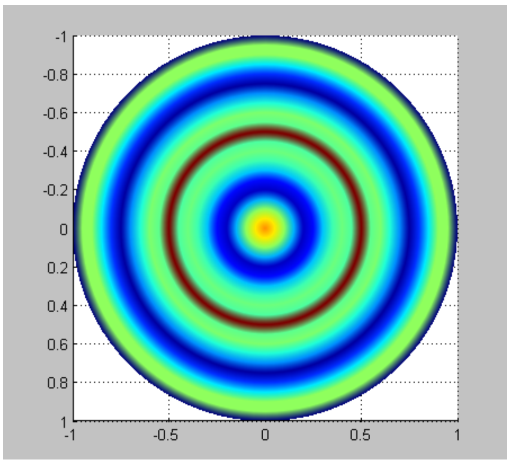

Figure 4a full Polar representation

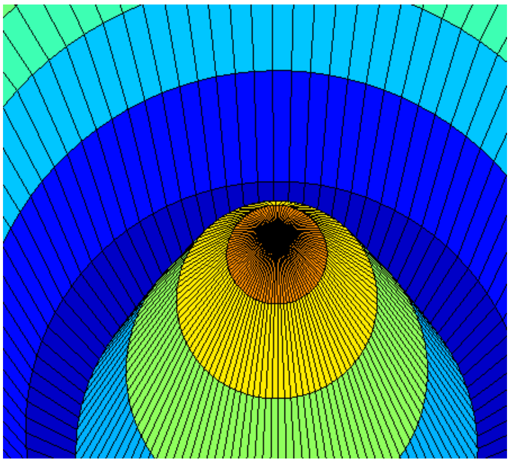

Figure 4b Close up of centre node

Now that we have the ability to interpret the polar mesh we can look further into analysing the problem of the centre node. Figure 4a, shows a polar representation of the flat mesh in Figure 3. There is an excitation line around the circle shown ten nodes from the centre. Figure 4b, shows a close-up of the polar mesh at the centre point. This shows that there is no break in the 'folding symmetry'. It also highlights the centre circle mentioned earlier, which is equivalent to the 101 nodes length, 'y' direction, of the square mesh in Figure 3, along 'x=0'line . Figure 5, shows zoomed out polar mesh in a 3D Matlab plot.

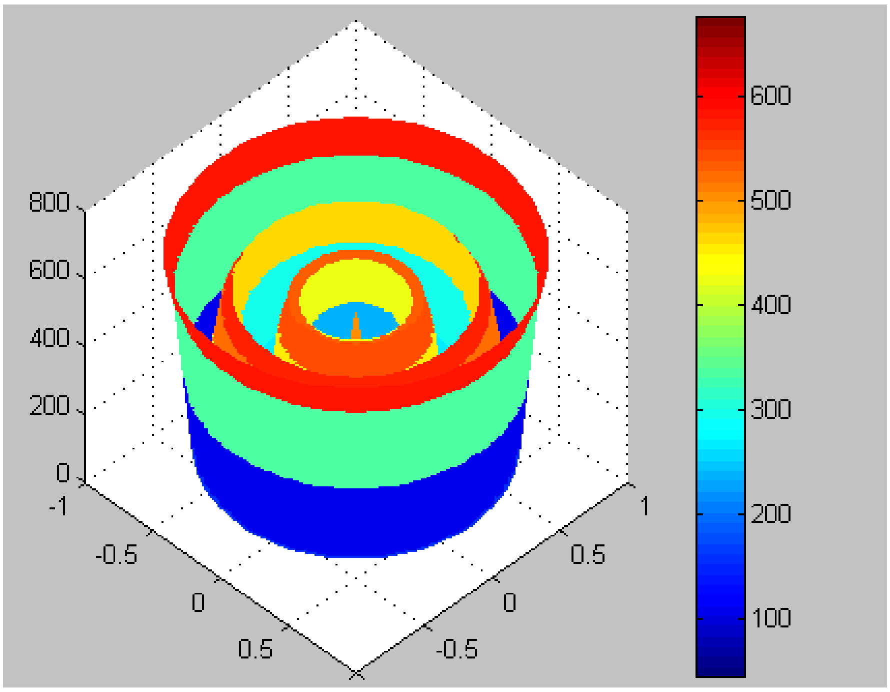

Figure 5 View of polar mesh, showing r, θ and magnitude

The Centre Node

As we have seen earlier in the paper the method used at the centre of the polar mesh is a circle of infinitesimal circumference. This circle is considered as short-circuit or open-circuit depending on the mode choice [2]. This is an unreasonable restriction which means that certain modes and their corresponding pressure/displacement fields cannot be accurately represented.

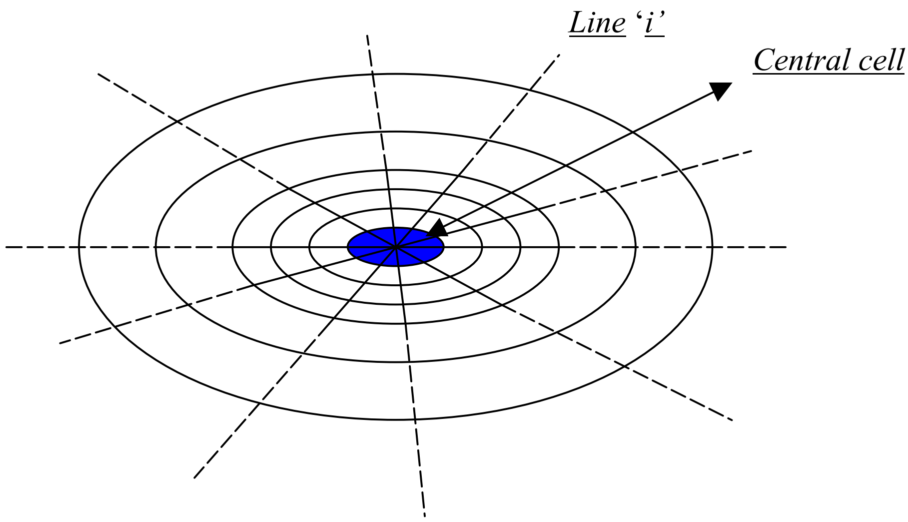

Figure 6 An orthogonal polar mesh

Figure 6 shows an orthogonal polar mesh. In the treatment by Cacoveanu et al [1] the centre of the mesh is a node with $2n$ branches and an open circuit stub, (not shown). The $2n$ branches ensure the continuity of the radial lines, while the stub ensures the synchronism of impulses. The central node is realised as a shunt connection of ideal lossless transmission lines. The nodal scattering matrix for this node can be determined in the same manner as all other physically realisable TLM nodes.

A voltage impulse travelling along line *'i'* will see an impedance discontinuity at the junction of the transmission lines. If the admittance of the branches is y and the admittance of the open circuit stub is *y/s,* then the voltage reflection coefficient is given as:

$$\Gamma = -\frac{2(n-1)y + y_s}{2ny + y_s} \tag{3}$$

and the transmission coefficient for each of the $2n - 1$ branches is given as:

$$\mathrm{T} = \frac{2y}{2ny + y_s} \tag{4}$$

The admittances y and *y*/*s*, are calculated in the same manner for all the other nodes in the mesh. The total inductance of each branch of the central node is :

$$L = \frac{\mu w}{2}\left(\frac{\theta}{2} + \cot\frac{\theta}{2}\right) \tag{5}$$

where θ is the angle between two adjacent branches and w the cell dimension in the z direction. For the central node, the axial field component and transversal field, for the Φ direction, can be determined following the procedure for an irregular mesh. The central cell dimension for the Φ direction is 2 $r\sin(\theta/2)$.

Cacoveanu et al [1] proved that the centre node of a polar mesh can be seen as a cell where all the nodel points intersec This method can therefore be applied to the polar representation discussed earlier in this paper. The intersection point is therefore made from one side of the rectangler mesh, the scatter and connect at what would be oposite nodes are allowed to connect to each other. For example, for a ten node length mesh space the scatter and connect between the nodes would be 1-6,2-7,3-8,4-9, etc. back to 10-5. This forms what would have been a central single point into a cross propagation point, from this we have a single cell. For this example ten nodes in size. At the point of inter section the stubs at the nodal cell point are given the reflection coefficient described above. This method then creates the polar mesh central node characteristic, which complete the representation of polar TLM.

Conclusions

The method described in this paper has allowed the author to apply polar TLM to acoustic problems, where pipe networks are required, and gives an accurate representation of polar problems within the ease of normal rectangler TLM programming. Further studies are under way to develop interfacing of these simplified TLM polar representations to 3D applications.

References

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