A matched absorbing node in TLM models

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Abstract

We show how it is possible to construct a broad-band absorbing node which can be incorporated within TLM shunt-node networks to form matched, distortionless boundaries in one- and two-dimensional formulations.

Introduction

The simulation of wave propagation processes generally involves a solution of the wave equation. This can be done analytically for simple conditions, but with realistic inputs/boundaries it is more convenient to use a numerical technique. The wave equation can be expressed in a finite difference form which can then be solved iteratively. An alternative approach involves replacing the wave problem by an electrical circuit analogue which can then be solved numerically or otherwise. Transmission Line Matrix (TLM) is one such analogue technique where signals propagate across a network of transmission lines. In addition to discretising space transmission lines discretise time by virtue of the inherent propagation delays which they create. Signals propagating through a transmission line network follow the wave equation and thus a solution of the currents and voltages in the network provides an instantaneous solution of the wave equation in terms of the Maxwell field components. Christopoulos [1] provides a very useful introduction to the technique.

Electromagnetic and acoustic modellers frequently have to deal with scattering problems which are open or unbounded. In order to maintain the computational load within reasonable limits many modelling techniques require that some form of artificial boundary must be invoked. This should provide a perfect match between the immediate problem space and its surrounds. Various methods methods for achieving this within finite difference formulations have been proposed and are contrasted in the next paragraph.

In the transmission line matrix (TLM) technique this might be approached by means of a reflectionless ($\rho = 0$) boundary description. However, it is well known that this is not appropriate over a wide range of frequencies and angles of incidence. To overcome this

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difficulty a number of options have been investigated and the status as at 1995 has been reviewed by Chen [2]. The Discrete Green's Function (Johns Matrix) [3] stores the response at the boundary nodes due to all the region beyond the boundary. A variant of this technique has also been proposed for heat-flow modelling in open-boundary problems [4]. Eswarrappa et al [5] have used a technique whereby the region outside the immediate problem space becomes progressively lossy. By far the most popular approach to perfect matched load (pml) boundary modelling is to use either the Higdon [6] or Berenger [7] boundaries. These are essentially finite difference descriptions which are bolted onto the periphery of a TLM model. Although they work effectively, they largely ignore the intuitive essence of TLM as promoted by Johns himself. The nearest approach to this philosophy is the method used in acoustic propagation by O Connor [8]. This assumes that a proportion of every impulse is absorbed (in a method which is not specified) at every node within a boundary layer at every iteration.

In this paper we attempt to provide a similar physical basis for perfectly matched layers by constructing a network of broad-band absorbing TLM nodes. Initially we will concentrate on a single broad-band, matched, distortionless absorbing TLM node. Our approach is to start by developing a node which attenuates voltage impulses. We will show that the degree of attenuation is critically dependent on the matching of several parameters. We will then outline how the correct choice of parameters lead to an identical attenuation for current. The paper will conclude with a demonstration of how this simple network might be applied in two- and three-dimensional models.

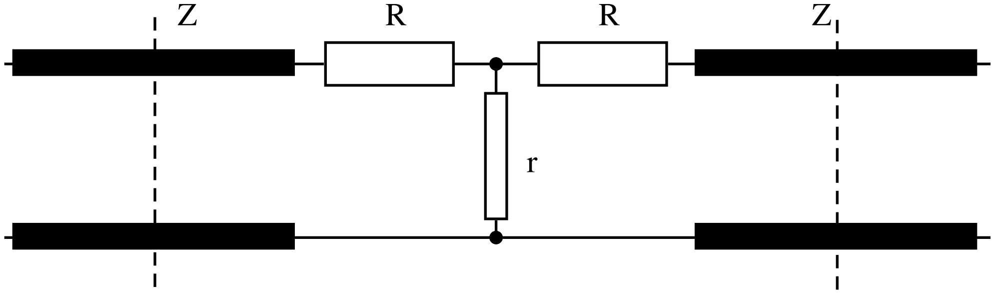

Figure 1 A diffusion node with a shunt loss

Theory (I)

Conditions for matching

The standard lossy TLM node comprising a series connection of transmission lines and resistors introduces attenuation and phase shift. Additional degrees of freedom can be introduced by inclusion of a shunt impedance. In the simplest case this can be a shunt resistor as shown in figure 1. We can then arrive at a matching condition that applies to a voltage impulse which is travelling along the transmission line on the left side of figure 1.

$$Z = R + \left[\frac{1}{r} + \frac{1}{R+Z}\right]^{-1} \qquad (1)$$

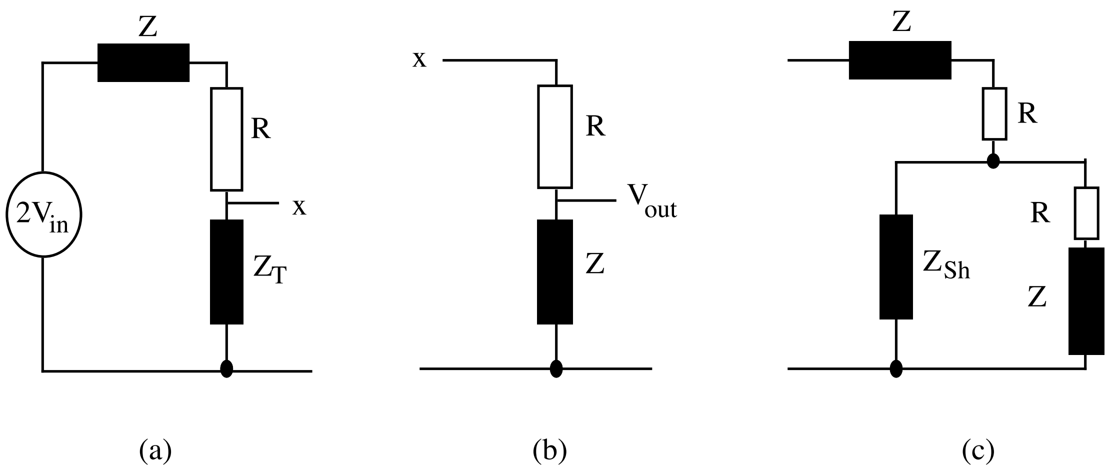

Figure 2 (a) a generalised representation of the circuit in figure 1. (b) the Thévenin equivalent circuit

Provided this condition applies then an incoming signal will not be reflected at the end of the left-hand transmission line. It will however be attenuated by the network and the extent of this can be estimated as follows. The Thévenin equivalent circuit for the entire network of figure 1 is shown in figure 2(a) with all the parallel impedances lumped into one, Z_T. The matching condition is then given by:

$$R + Z_T = Z \quad \text{or} \quad Z_T = Z - R \qquad (2)$$

The potential at point x due to a single incoming pulse, V_{in} in the Thévenin circuit is given by:

$$V(x) = \frac{2V_{in} Z_T}{R + Z + Z_T} \qquad (3a)$$

If the matching condition is applied then this becomes:

$$V(x) = \frac{2V_{in} (Z - R)}{2Z} \qquad (3b)$$

The voltage which moves into the output transmission line can then be obtained from a potential divider analysis based on figure 2(b):

$$V_{out} = V(x) \frac{Z}{Z + R} = V_{in} \frac{Z - R}{Z + R} \tag{4}$$

Thus we have an attenuation factor F_V ($0 < F_V \leq 1$) given by:

$$F_V = \frac{V_{out}}{V_{in}} = \frac{Z - R}{Z + R} \tag{5}$$

Although any choice of R will provide an output which is attenuated this analysis gives no consideration for the preservation of the character of the impulse. In order to be consistent with the equivalent 'infinite' lossless TLM mesh which this network is supposed to bound it is necessary that it does not introduce dispersion. The next section investigates the conditions that provide distortionless propagation.

The distortionless condition

Dispersion of an impulse can be inhibited if the network is adapted so that it fulfils the Heaviside condition for a distortionless line. For a line with inductance, L, capacitance, C, resistance, 2R and conductance, G the condition is

$$\frac{L}{C} = \frac{2R}{G} \tag{6}$$

For the node in figure 1 with a characteristic impedance, Z and conductance, G = 1/r this yields

$$Z^2 = 2R\,r \tag{7}$$

The integration of distortionless and matching conditions

Equation (1) simplifies to

$$\frac{r(2R + Z) + R(R + Z)}{R + Z + r} = Z \tag{8a}$$

With normalised characteristic impedance (Z=1) this reduces to

$$R^2 + 2rR - 1 = 0 \tag{8b}$$

The non negative root of this equation is

$$R = -r + \sqrt{r^2 + 1} \tag{8c}$$

R can also be defined for the distortionless condition as

$$R = \frac{1}{2r} \tag{8d}$$

A plot of R vs r for eqns 8c and 8d show that they never coincide for $r<\infty$. This is easier to see if eqns 8b and 8c are solved in terms of r:

$$r = \frac{1-R^2}{2R} \text{ (matching)}, \; r = \frac{1}{2R} \text{ (distortionless)} \tag{8e}$$

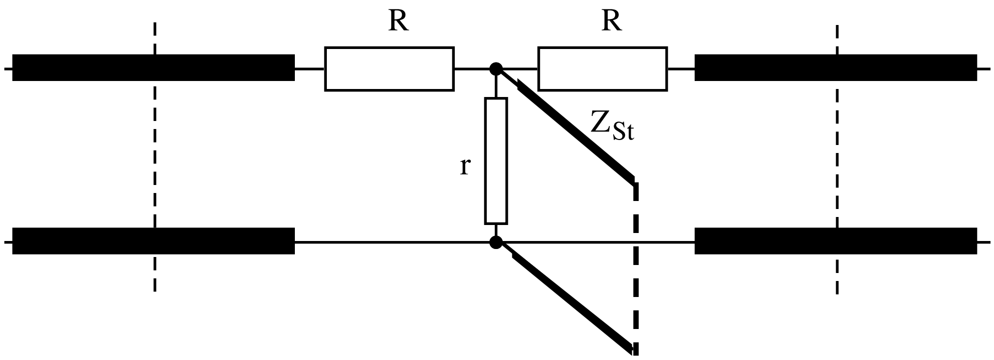

Figure 3 Lossy diffusion node with capacitive stub

A revised network

This situation can be radically altered by the inclusion of a stub transmission line which is placed at the node centre as shown in figure 3. Using the standard conventions in TLM the initial choice is an open circuit terminated half-length stub.

The inclusion of the stub means that the total capacitance of the node is increased and the Heaviside condition (assuming normalised impedances) now becomes

$$\left[\frac{Z_{St}}{Z_{St} + 1}\right]^2 = 2R\,r \tag{9a}$$

$$R = \frac{1}{2r}\left[\frac{Z_{St}}{Z_{St} + 1}\right]^2 \tag{9b}$$

The revised matching condition is

$$\frac{rZ_{St}(2R+Z) + RZ_{St}(R+Z) + rR(R+Z)}{Z_{St}(R+Z+r) + r(R+Z)} = Z \tag{10a}$$

When this is solved for R the positive root is:

$$R = \frac{-r Z_{St} + \sqrt{r^2 Z_{St}^2 + Z_{St}^2 + 2 r Z_{St} + r^2}}{r + Z_{St}} \qquad (10b)$$

Plots of eqns (9b) and (10b) vs r for the case where $Z_{St} = 2$ are shown in figure 4 and confirm that there is a point of coincidence which can be determined iteratively as r = 0.28432.

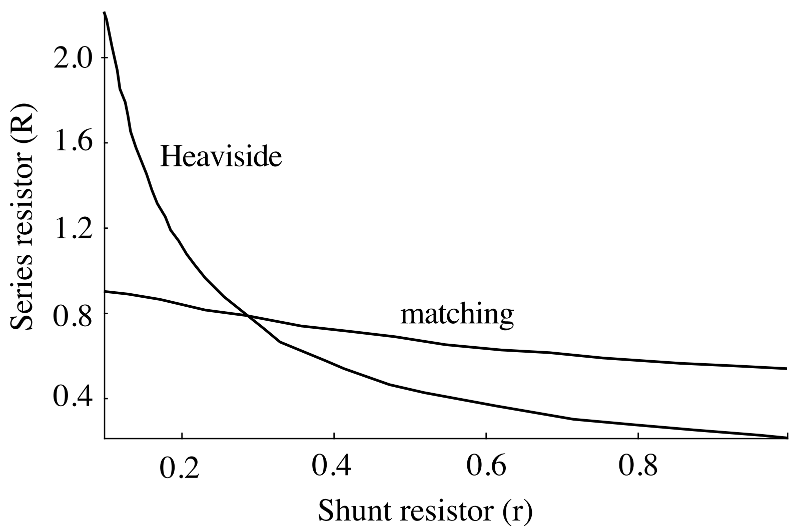

Figure 4 Plots of the matching and distortionless dependence of R as a function of r for a node with a capacitive stub $Z_{St} = 2$.

Spurious dispersion

The response of this node to an input signal did indeed indicate proper attenuation, but there was evidence of dispersion which was dependent on the frequency components of the input [9]. The reasons for this are in fact straight-forward. The open-circuit, half-length stub in TLM is an approximation to an additional capacitance, but carries with it spurious inductance. Similarly, a short-circuit, half-length stub approximates additional inductance, but carries spurious capacitance. In fact neither of these on their own are suitable for the Heaviside condition (eqn (6)). The only suitable arrangement which satisfies this via eqn (9) is to have an infinite stub, so that there is no reflected signal. Thus we have Z_{St} with $\rho_{St} = 0$. When this is applied there is no error inductance or capacitance.

Initial tests on the final revision

Some practical tests of propagation on a one-dimensional TLM network with a boundary of distortionless matched nodes with 'infinite' stubs was found to introduce no dispersion effects. However it was noted that the return signal was very sensitive to the precise coincidence of the matching and distortionless conditions. An input pulse of magnitude 1000 was allowed to traverse a one-dimensional lossless mesh before interacting with a succession of absorbing nodes each with $Z_{St} = 0.1$ ($\rho_{St} = 0$), R = 0.99602 and r = 0.004149. The return signal was inverted with magnitude 2×10^{-2} and displayed no dispersion. When the node was defined with greater resolution (R = 0.99602447204 and r = 0.00414872477) the return signal was a perfect copy of the original, but with an amplitude of 1.75×10^{-13}.

Thus, we now have the parameters, R and r which correspond to a particular Z_{St} and which provide matched distortionless attenuation. The magnitude of the attenuation factor, F for a single node can be obtained using eqn (5). For example, in the case of $Z_{St} = 0.1$ we have F = 0.00199 and this result is consistent with what was observed in the experimental simulation described above.

Theory (II)

The conditions for a wideband current absorber

The attenuation analysis which was given in section 2.1 can be repeated for the incoming current pulse. A current, I_{in} which arrives at point x in figure 2(c) subsequently splits between the two paths. The relative magnitudes of these currents depend on the impedances which they encounter. Thus the current, I_{out} which travels through the output transmission line is given by:

$$I_{out} = I_{in} \frac{Z_{sh}}{Z_{sh} + R + Z} \tag{11}$$

Z_{sh} is the shunt impedance represented by the parallel combination of stub, Z_{St} and resistor, r. When the substitutions are made for Z_{sh} we can express the ratio of I_{out} to I_{in} as:

$$F_I = \frac{rZ_{St}}{rZ_{St} + rR + RZ_{St} + rZ + ZZ_{St}} \tag{12}$$

In the next section we will demonstrate by means of examples that $F_I = F_V$ when the matched and distortionless conditions are applied.

Table I

ZSt	r	R	F_V	F_I
1000	5.702883	0.087499	0.83908	0.83908
500	4.417292	0.112740	0.79736	0.79736
100	2.378235	0.206097	0.65824	0.65824
10	0.846883	0.879339	0.34410	0.34410
1	0.14144584	0.88373046	0.06172	0.06172
0.1	0.00414872	0.99602447	0.00199	0.00199

Applications and Discussion

The match and distortionless conditions can be applied for a range of values of stub impedance to investigate the relationship between the balance conditions which are required for different levels of attenuation. These are summarised in Table I where F_V is calculated using eqn (5) and F_I is calculated using eqn (12). It can be seen that there is no difference between the current and voltage attenuation factors. A plot of F vs Z_{St} is shown in figure 5 a represents the attenuation which a signal will experience when it interacts with a single matched distortionless line. These results were compared with the transfer function in a practical simulation. A one-dimensional network consisiting of 20 losless TLM nodes was constructed. A single matched/distortionless node was placed at the centre of this network. A right-moving step function input of magnitude 1000 was injected at node 1 and the potential at nodes 12 and beyond was observed. The output was a step-function with reduced magnitude. The ratio V_{out}/V_{in} was found to be identical with the calculated values of F for any Z_{St}. The results also agree exactly with the value calculated using eqn (5) if the value of R which provides matching at that Z_{St} value is inserted.

A single attenuating node can placed at the centre of link-lines in a two-dimensional TLM network as shown in figure 6. Under normal circumstances such constructions are not advised, because parity mismatch can lead to mode mixing and spurious effects. However, since our node is perfectly matched, there is no spurious return signal and therefore no mode mixing. Figure 7 shows the transition between a lossless region and a matched-load boundary using our attenuating network. One thing that is easily overlooked is the loading effect of the stubs which are used within the attenuating region. This has the effect of slowing down a signal. In order to maintain algorithmic simplicity it is possible to have an entire network with conventional stubs and attenuators as shown in figure 8. Within the lossless region $F_A = 1$ and Z_A is chosen so as to maintain equal loading across the entire network. Within the absorbing boundary $F_B < 1$ and $Z_B \sim \infty$. By this means it is possible to maintain perfect propagation across the network, even into the attenuating region.

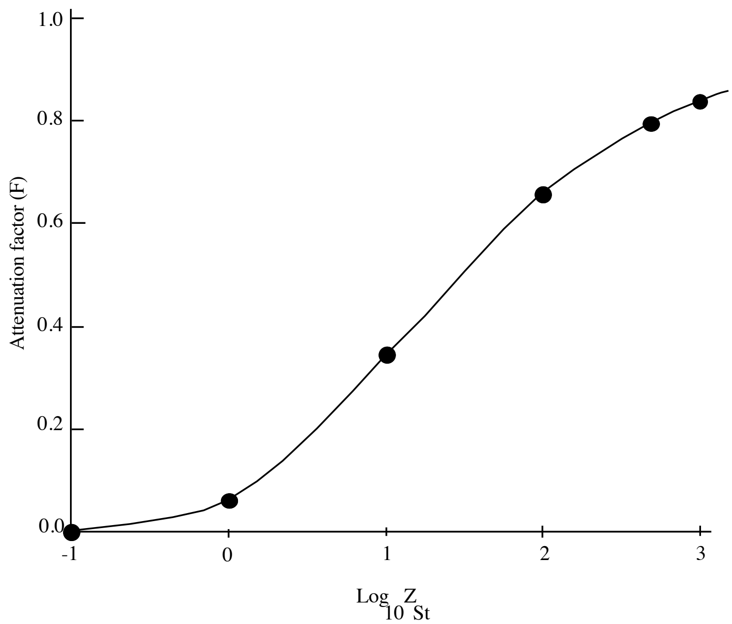

Figure 5 A plot of attenuation faction, F against Z_{St}, where Z_{St} is the complementary value which guarantees the matched/distortionless conditions for particular values of R and r.

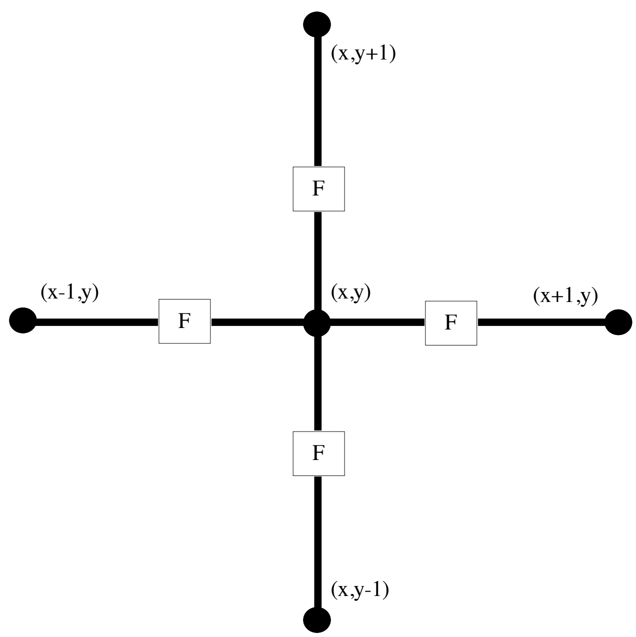

Figure 6 The arrangement of attenuating networks (designated 'F') at the centre of the lines linking adjacent two-dimensional nodes.

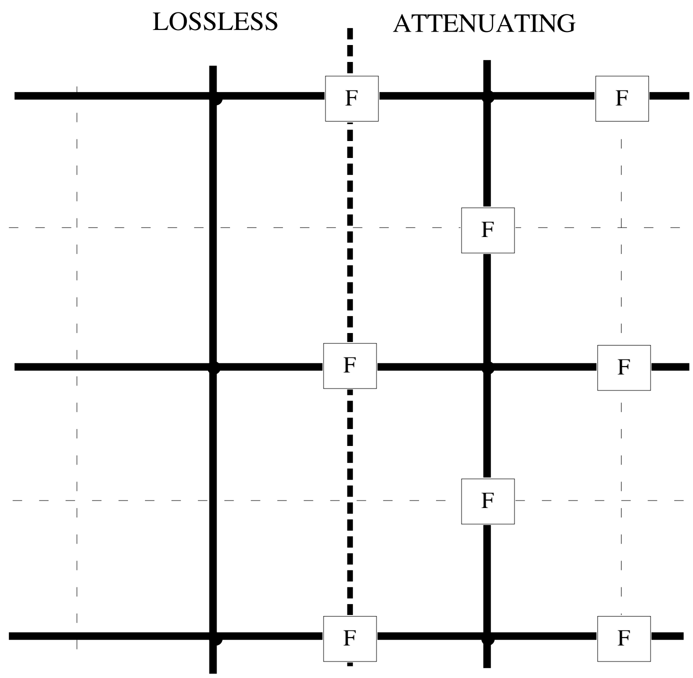

Figure 7 The interface between lossless and attenuating regions in a two-dimensional TLM network.

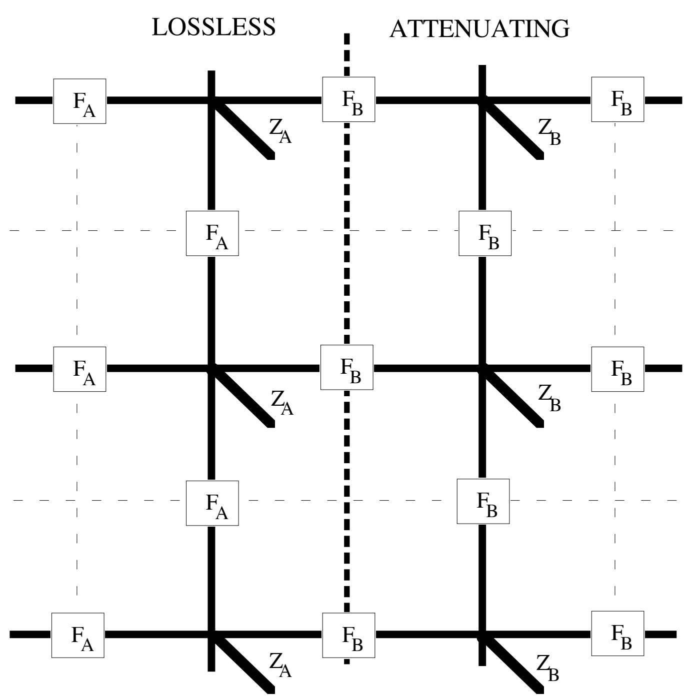

Figure 8. Fully stub-loaded mesh arranged for algorithmic simplicity. Since region A is lossless $F_A = 1$ and Z_A is chosen so as to maintain equal loading (velocity) across the entire network. Within the attenuating region we have $F_B < 1$ and Z_B can be set equal to infinity.

Conclusions

This work has shown that a TLM formulation of a broad-band attenuator for use as part of an absorbing boundary is trivially simple, although the analysis which was used to confirm the basis for its use has demonstrated some curious features such as the need for perfect matching of parameters and the effective loading of the mesh which should be appropriately compensated in the lossless regions being modelled. Although, it has been developed for TLM in one and two-dimensions, the concept has much wider application and the computational simplicity of this pml should lead to a significant improvement in the efficiency of numerical models for wave propagation.

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