The time-domain dynamics of 2-D TLM

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Abstract

This paper revisits the fundamentals of the simplest, 2-D TLM scalar modelling. Separate voltage and current distribution speeds are defined, leading to novel derivations and understanding of the wave speed and dispersion, wave impedance, and mesh inductance and capacitance. The ability of TLM to achieve the necessary impulse response is reviewed, and TLM wave speed and dispersion characteristics are reconsidered in this light. The focus is on gaining insight into well-known TLM results while a) remaining entirely in the time domain and b) explicitly relating all "macro" results directly to the simplest, elementary, single scattering event at the "micro" level.

1 Introduction

At the heart of the scalar 2-D TLM algorithm is the simple scattering shown in figures 1a and 1b.

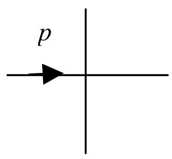

Figure 1a

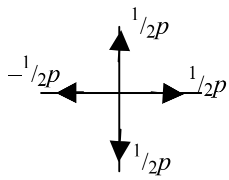

Figure 1b

How this "simple" scheme can represent wave phenomena has been extensively studied. The associated wave speed, dispersion relations, and so on, have been established using techniques based on circuit theory, linear analysis (matrix eigen-value problem formulation), and properties of periodic structures. The results of these approaches are usually presented in the frequency/wave domain.

The time-domain performance of the algorithm, meanwhile, remains somewhat unexplained. Some intriguing questions deserve more direct attention. For example:

- A question that must strike every newcomer to TLM is: If the pulses travel at $\Delta l/\Delta t$, and if the only thing that happens to them is the repeated application of Fig.1, where exactly, in the time domain, does a wave speed of $\sqrt{^1/_2}\,\Delta l/\Delta t$ come from? Can figure 1 alone "explain" a) the slow wave speed and b) the factor of precisely $\sqrt{^1/_2}$, particularly for wave propagation in the axial direction?
- A waveform in TLM is made up of a series of pulses, like figure 1a, each of which is continually "dispersing" (spreading out) further and further with every

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time step. This is evident from repeated application of Fig.1. Yet somehow each pulse must also retain some "coherence", as otherwise wave propagation would not be possible, not even at very long wavelengths, as all crests and troughs would gradually merge into each other. So, which is it? continuous dispersion? or retention of coherence? And if (as must be the case) the pulses somehow do *both*, what exactly is the mechanism in the time domain?

- TLM modellers often excite their model not with a single impulse but with a Gaussian distribution of pulses (a series of pulses whose envelope in space has the shape of a Gaussian curve). This cluster of pulses are then found to propagate without dispersion. The explanation of this coherence in the *frequency* domain is familiar: the problematical high frequency components in the "impulse" have been removed: the low frequency components then propagate without dispersion. But again, each component pulse in the Gaussian envelope is (as it were) "unaware" of its neighbours, "unaware" that it is part of a wider Gaussian envelope. Further each of these component pulses is "dispersing" indefinitely. So what is the time-domain method by which the overall distribution retains its coherence? In other words, how does each of the component pulses independently maintain the correct speed and shape to preserve the overall shape?

Somehow the scattering of figure 1 must tell the whole story: wave speed, dispersion, apparent impedance, capacitance and inductance, …and all without leaving the time domain.

Throughout what follows, a pulse, such as p in Fig.1a, represents a voltage pulse in a shunt TLM network, with an associated current pulse, p/Z_0, where Z_0 is the characteristic impedance of the link lines. The pulse speed is the ratio of the length of the TLM link line to the model time increment, or $\Delta l/\Delta t$. All the arguments that follow are easily adapted to apply to series networks as well.

2 Extracting "speeds" from figure 1

For pulses as in Fig.1 the usual definitions of wave speeds don't work, so a new definition for "voltage distribution speed" is proposed, the horizontal or x-direction component of which is defined as

$$(c_v)_x = \frac{1}{\Delta t}\left[\left. \frac{\sum v_i x_i}{\sum v_i} \right|_{t+\Delta t} - \left. \frac{\sum v_i x_i}{\sum v_i} \right|_{t} \right] \tag{1}$$

where v_i are the values of all TLM voltage pulses, x_i are the horizontal distances of these pulses measured from some fixed vertical axis, and the first ratio of summations is carried out one time increment, Δt, later than the second, perhaps when the pulses are midway between nodes, $^1/_2\ \Delta t$ after a scattering event. In words, Eqn.1 says that the component of the speed in the x-direction is the change in the x-coordinate of the "centre of gravity" (geometrically weighted centre) of the entire voltage distribution in the time increment Δt. The units are m/s.

Note that this definition directly and seamlessly links the "micro" and the "macro" TLM propagation speed behaviour. It does so without having to introduce standard "wave" concepts such as phase, wavelength or wave number. It works properly for a single original pulse or for the result of many pulses superposed. Also it can be applied locally or it can be extended over an arbitrary number of nodes, to provide either a highly localised "wave" speed or an average "wave" speed over an arbitrary area. The only proviso is that the summations be carried out over the "same" pulses before and after scattering.

As the entire sum of voltage pulses $\sum v_i$ is conserved over time by the TLM algorithm (not just in going from figure 1a to 1b but for all subsequent time), this $\sum v_i$ term can be removed from the time differential in Eqn.1, giving an alternative definition of the voltage speed as

$$(c_v)_x = \frac{1}{\Delta t \sum v_i}\left[\sum v_i x_i\Big|_{t+\Delta t} - \sum v_i x_i\Big|_t\right] \tag{2}$$

A similar definition applies to the voltage speed in the vertical or y-direction.

"Current speed" can similarly be defined as

$$(c_i)_x = \frac{1}{\Delta t}\left[\frac{\sum i_i x_i}{\sum i_i}\Bigg|_{t+\Delta t} - \frac{\sum i_i x_i}{\sum i_i}\Bigg|_t\right] \tag{3}$$

in the x-direction, with a similar definition for the y-direction. Note that current is a directed quantity, so that, for example, a negative pulse travelling in the negative direction becomes positive.

These definitions are now applied to figure 1. Taking the initial horizontal position of the pulse in Fig. 1a to be x_0 from some arbitrary reference vertical axis, the voltage speed in the x-direction by this definition is

$$(c_v)_x = \frac{\left[(-\tfrac{1}{2}p)x_0 + 2(\tfrac{1}{2}p)(x_0 + \tfrac{1}{2}\Delta l) + (\tfrac{1}{2}p)(x_0 + \Delta l)\right] - \left[(p)(x_0)\right]}{\left[-\tfrac{1}{2}p + \tfrac{1}{2}p + \tfrac{1}{2}p + \tfrac{1}{2}p\right].\Delta t} \tag{4}$$

which reduces to

$$(c_v)_x = \frac{\Delta l}{\Delta t} \tag{5}$$

Similarly, the current speed in the x direction is

$$(c_i)_x = \frac{\left[\frac{1}{Z_0}\right]\left[-(-\tfrac{1}{2}p)x_0 + 2(0)(x_0 + \tfrac{1}{2}\Delta l) + (\tfrac{1}{2}p)(x_0 + \Delta l)\right] - \left[\frac{1}{Z_0}\right]\left[(p)(x_0)\right]}{\left[\frac{1}{Z_0}\right]\left[-(-\tfrac{1}{2}p) + \tfrac{1}{2}p\right]\Delta t} \tag{6}$$

or

$$(c_i)_x = \frac{1}{2}\frac{\Delta l}{\Delta t} \tag{7}$$

In other words, the voltage distribution is advancing at *twice* the speed of the current distribution. (In figure 1 both voltage and current speeds in the vertical direction are zero.)

Furthermore, if the "connect" and scattering in figure 1 are continued and these definitions are applied to subsequent time steps, the two speeds remain constant indefinitely: the voltage distribution speed stays at exactly twice the current distribution speed for all future time.

3 A second pulse needed?

But pulse distribution speed is not exactly the same as wave speed. Waves cause energy to travel, and for this to be possible (at least in a lossless way), the energy must be continuously transformed from one form (e.g. voltage or electrical) to another (e.g. current, or magnetic). This in turn means that, as the wave propagates, the voltage energy and the current energy must advance at the same speed. Furthermore, on average, the amount of energy of each kind must be equal, which in turn implies that the proportion between voltage and current must remain constant. This speed is the wave speed, and the proportion is the wave impedance.

So, if TLM allows waves to propagate, somehow it must *reduce* the voltage distribution speed (Eqn.5) and *increase* the current distribution speed (Eqn.7) until they are equal. The simplest way to achieve both requirements is by superimposing on figure 1a a pulse in the opposite direction, as shown in figures 2a and 2b. An exploration of this approach will be considered first. (Note that such a second, counter-propagating pulse would be needed in any case if it were required to set up an arbitrary initial voltage and current "pulse" in figure 1a), unless their ratio happened to be the link line impedance Z_0.)

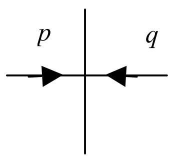

Figure 2a

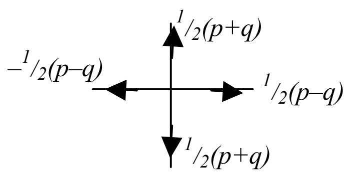

Figure 2b

If the same definitions are applied when two pulses are present, the following voltage and current distribution speeds are obtained:

$$\frac{\left[(-\tfrac{1}{2})(p-q)x_0 + 2(\tfrac{1}{2})(p+q)(x_0 + \tfrac{1}{2}\Delta l) + (\tfrac{1}{2})(p-q)(x_0+\Delta l)\right] - \left[(p)(x_0) + (q)(x_0+\Delta l)\right]}{[p+q]\Delta t} \tag{8}$$

or

$$(c_v)_x = \left[\frac{p-q}{p+q}\right]\frac{\Delta l}{\Delta t} \tag{9}$$

Similarly

$$(c_i)_x = \left[\frac{1}{2}\frac{(p+q)}{(p-q)}\right]\frac{\Delta l}{\Delta t} \tag{10}$$

If these are to be equal, the following must apply:

$$\left[\frac{(p+q)}{(p-q)}\right] = \sqrt{2}\,. \tag{11}$$

This in turn implies that

$$(c_v)_x = (c_i)_x = \sqrt{{}^1\!/_2}\ \Delta l/\Delta t. \tag{12}$$

In other words, if the voltage and current distributions are to have the same speed, somehow counter-propagating pulses must exist, and be of just the right relative magnitude. If so, the only possible common speed for the two component waves (and so for the entire wave) is $\sqrt{{}^1\!/_2}\ \Delta l/\Delta t$. If this ratio is not quite right, the wave cannot continue to propagate without "breaking up" over time, due to the inability of the voltage and current distributions to advance together with a fixed proportion between them, as required.

The same applies to movement in the y-direction, so that the overall movement of the voltage and current pulse distributions (or waves) will be

$$\underline{\boldsymbol{c}} = c_x\underline{\boldsymbol{i}} + c_y\underline{\boldsymbol{j}} \tag{13}$$

where underlined quantities are vectors, and $\underline{\boldsymbol{i}}$ and $\underline{\boldsymbol{j}}$ are unit x and y vectors.

4. Mesh impedance, inductance and capacitance

Note that, by introducing a second, counter-propagating pulse and determining its required magnitude, at least three other properties of the TLM model have been established implicitly, also in a novel way (to the author's knowledge). These are the impedance, inductance and capacitance of the modelled wave (or of the mesh).

Thus, in figure 2, the voltage during scattering at the node will be ${}^1\!/_2(p+q)$, while the current in the x-direction before and after scattering will be $(p-q)/Z_0$. Under these conditions, necessary for continued wave propagation, the "mesh" (or "wave") impedance, Z_m, defined as the ratio of voltage to current for a freely propagating wave, will be

$$\begin{aligned} Z_m &= Z_0\,{}^1\!/_2(p+q)/(p-q) \\ &= \sqrt{{}^1\!/_2}\ Z_0. \end{aligned} \tag{14}$$

The wave speed and wave impedance together implicitly define the "mesh" distributed capacitance and inductance per meter as

$$\begin{aligned} C_m &= (1/Z_m)(1/c) \\ &= 2\ \Delta t\ /\ Z_0\Delta l \end{aligned} \tag{15}$$

$$L_m = Z_m/c$$

$$= Z_0\ \Delta t\,/\,\Delta l \qquad (16)$$

all agreeing with the results derived by other methods.

5. Questions about the *second* pulse

But, it might be objected, the initial thesis was that all the wave properties must be present in the scattering of a single pulse (Fig.1). Yet, to make progress, a second pulse had to be introduced, and of precisely the right size. Where does this second pulse come from? What happens if it is not there? Does it need to stay close to the first pulse as it propagates? If so, how does it do it? Finally, as there are now *two* initial pulses continuously spreading out in space, rather than the original single pulse, what about the subsequent ongoing "dispersion" at the micro level?

Regarding the source of the second pulses, one obvious answer is that, in a TLM model, they come from the initial conditions, or from the boundary conditions, or from both. Thus, if a TLM scheme is initialised with a propagating wave, there will have to be an initial voltage and current specified, with the correct ratio between them. Alternatively, if the field is established by boundary conditions, these will supply the necessary counter-propagating pulses. The TLM mesh will "tell" the boundary, via the "apparent" (or mesh) impedance, what pulses should be fed into the mesh so that the correct ratio between pulses in opposite directions is established.

But again, what about a single pulse, on its own, in the middle of a mesh? It must somehow find, or generate, its own counter-propagating pulse, and keep this pulse moving with itself, without any help from initial or boundary conditions. So more explanations are needed.

Then there is the question of dispersion. The term "dispersion" is frequently understood to refer to a wave speed changing with wavelength (or frequency). But in a looser sense it can be applied to the way a pulse "disperses", or spreads out, or loses coherence. The two concepts are, of course, closely related.

In 1-D TLM, there is no dispersion in either sense: a voltage pulse has an associated current pulse, always exactly proportional to it, and both pulses advance at $\Delta l/\Delta t$, through the "scattering" and "connect" parts of the algorithm. In 2-D TLM however, at every scattering event pulses become dispersed, voltage and current distributions separate, and the "impedance" becomes unclear. Above it was assumed that p and q were exactly "right" and that their effects were superposed in space. In fact, all the analysis so far will work even if, instead, pulses p and q are far apart. And even if it is assumed that p and q are incident on the same node as in Fig.2, it is clear that at the next time step some things will begin to go wrong. The pulses are travelling in opposite directions, and so the "centre of gravity" of p is separating from that of q at a rate of $2\Delta l/\Delta t$, even though the combined "voltage distribution speed" as defined above will still be correct.

Once again, more explanations are needed.

6. Returning to the single pulse

In fact, wonderfully, the single pulse can do all that is required of it, on its own. In fact, it must do it "on its own" even if it is mixed in with a multitude of other pulses, because linear superposition applies. Putting it colloquially, each component pulse is unaware that it is (or is not) part of a wave and, if so, what the wavelength is. (Equivalently, neither does it know if it is part of a Gaussian distribution, nor where in the envelope it lies). The micro response of a single pulse, over time, must therefore explain the macro wave response completely, independently of other pulses.

Returning once again to figure 1, one can argue from the scattering pattern of the single pulse, without the second pulse of figure 2, to determine the "wave" parameters (Eqns. 12, 14, 15 and 16) directly as follows. A single pulse is propagating to the right in the link transmission line of distributed inductance and capacitance, the combination of which give a pulse speed $\Delta l/\Delta t$ and characteristic impedance Z_0. When it reaches the node, it sees a discontinuity, which must be capable of interpretation as a sudden change either in the line's inductance or in the capacitance, or in both. From this perspective what does figure 1 imply?

There is a sudden drop of the voltage by 50%, from p to $^1/_2p$, as if the charge in the distributed capacitance was shared equally with a second capacitor, in parallel, of equal capacitance to that of the line. Simultaneously, half the initial charge associated with p is temporarily delayed in its advance to the right. This is exactly the effect that would be seen if there was a sudden doubling of the capacitance at the node.

Yet the total current to the right is maintained, at exactly the level it would have had if there were no node present. In other words, none of the initial current p/Z_0 to the right has been diverted off the line, nor delayed. So the apparent line inductance has been preserved at the node to that of the link line.

Combining these observations, whereby the node has doubled the line distributed capacitance and left the distributed inductance unchanged, one can conclude that the wave impedance must be changed by a factor of $\sqrt{^1/_2}$ and the wave speed by the same amount. Thus the results expressed in Eqns. (12), (14), (15) and (16) are implicit in figure 1 alone.

Yet this seems to contradict the requirement of a second pulse, as argued above. To resolve this and the other apparently conflicting requirements, it is fruitful to observe again, in the light of all of the above, how a single pulse in fact behaves over time.

7. Response to a single pulse

Figure 3 shows the voltage distribution at two successive time steps corresponding to a unit pulse launched 22 time steps earlier. To keep matters uniform in the y-direction, rather than a simple pulse in one line, a column of pulses extending indefinitely in the y-direction was assumed. This makes the issue effectively one-dimensional, yet all the relevant effects of the 2-D mesh are still operative. Because of the assumed symmetry, the mesh can consist of a single line in the x-direction with half link lines in the y-direction, with reflection boundaries on the latter.

The form of this response, which is simply the result of repeated applications of figure 1, "explains" many of the issues raised. Yes, "dispersion" continues indefinitely, the whole distribution spreading further and further with each time step. But at the same time, somehow it retains a remarkable coherence, in the form of a relatively small "hump" which, despite some pulsating as it advances, consistently keeps its general shape. This hump, or cluster of pulses, therefore does not "disperse". Furthermore, it advances at exactly $\sqrt{^1/_2}\Delta l/\Delta t$, "explaining" the wave speed. It is spread over about 4 or 5 nodes in total. Ahead of the hump is a negligibly small voltage distribution. Behind the hump is a relatively high amplitude, short wavelength, oscillating system, which is quasi-stationary and decaying very slowly. It is not difficult to see how, for example, the convolution of this impulse response with a sinusoidal input will produce the main features of the familiar TLM wavelength-dependent wave speed.

But again, based on figure 1, how does it "work"? For example, how is the main feature, the all-important "hump", formed and maintained?

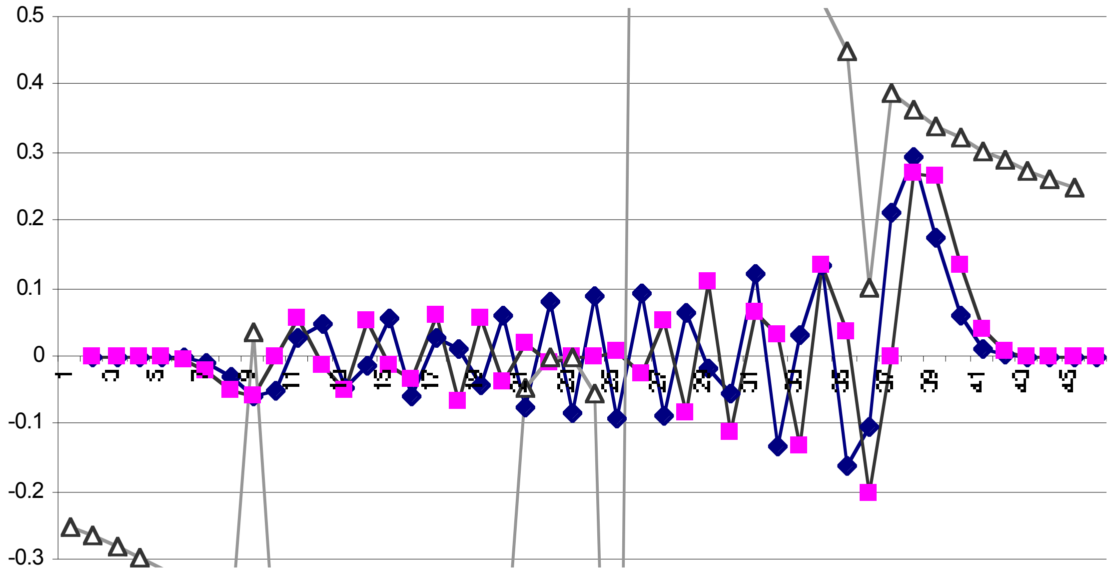

Figure 3. Response at 22 and 23 time steps after initialisation to a unit pulse originally at link 21, showing the cluster (or "hump"), in the region of links 37 to 42, advancing at $\sqrt{^1/_2}\,\Delta l/\Delta t$. The third curve shows the ratio of voltage to current (normalised with respect to Z_o) in the most advanced curve, scaled by 0.5 to fit on the graph. This "impedance" is of interest only towards the front of the waveform where it rises from 0.5 (current too lean with respect to voltage) towards 0.8 (current too rich). Behind the "hump" it oscillates wildly, eventually settling down at –0.5. The total current within the "hump" is equal to the entire original current of one unit, while the total voltage is $\sqrt{^1/_2}$.

8 The impulse response dynamics

In summary, the forming and maintaining of the hump is a highly dynamic, self-controlling or self-correcting system. At the front or leading edge, the situation is close to figure 1 in the sense that the local voltage speed is higher than the current speed, and the current distribution is slowing down the voltage. The wave is being starved of current. At the back of the hump the opposite is the case: there is an excess of current over voltage, tending both to speed up the local voltage distribution ahead, and, at the same time, to cut off the current behind, leaving the standing, oscillating system. Within the hump, the ratio of voltage to current rises from about $^1/_2 Z_0$ at the front (current too lean) to about $0.8Z_0$ at the back (current too rich), but the average ratio in the hump is the required $\sqrt{^1/_2}\, Z_0$. This means that the "average" ratio of p to q is exactly that required by Eqn.11, producing the exact wave speed $\sqrt{^1/_2}\, \Delta l/\Delta t$. Any tendencies to depart from this situation are automatically corrected by the workings of the scattering algorithm in figure 1.

In effect, the TLM algorithm of figure 1 has taken time and space to create a situation like Fig.2, with its counter-propagating pulses p and q, of just the right ratio (albeit a little spread out), but now propagating together at the same speed.

All this merits further analysis.

Consider figure 1 yet again. A single voltage pulse p drops, on scattering, to $^1/_2 p$. The associated x-direction current starts at p/Z_0 and remains at this value after scattering, but only by dint of sending back a negative voltage pulse. This process can now be seen in several equivalent ways.

a) As noted, the voltage speed is twice the current speed, so the voltage has gone too far ahead of the current, and TLM copes by sending back a negative pulse which
   - i) *Reduces* the average, or overall, voltage in this area;
   - ii) Immediately *increases* the current in this area;
   - iii) Creates a local, positive, spatial voltage gradient which will tend to produce a local, negative, temporal rate of change of current in subsequent time steps.
   - iv) Sends back a voltage signal, to (as it were) "send up more current from behind": thus, at the next time step, it will cause a positive, current pulse, at the previous node to the left, but travelling to the right, or forward.
   - v) This voltage pulse "asking for more current" travels at the full speed of $\Delta l/\Delta t$, as will its subsequent reflections, so eventually it can catch up again with the waveform that has gone ahead, provided the latter it is not getting away from it at full speed.

   In summary, Fig.1 shows at least the beginnings of a process of reducing (or slowing down) the advance of the voltage and increasing (or speeding up) the supply of current while also beginning to change the ratio of voltage to current to a more middling value between 0.5 (the value at the node, just after scattering) and 1.0 (the value in the link line, just before scattering).

b) Thinking in terms of impedances, one can say that apparent impedance, which in Fig.1 must be $^1/_2 Z_0$, is too low to preserve the voltage pulse. A low impedance means a drop in voltage without the corresponding drop in current.

c) As an alternative to thinking in terms of wave speeds and impedances, one can also think in terms of inductances and capacitances, even if ultimately they must be just two aspects of the same thing. Thus one can see in this process how

TLM is modelling increased distributed capacitance effects. There was not sufficient current in the original pulse to charge the distributed mesh capacitance, so the voltage fell and more current was needed and "requested".

At the very front of the waveform, repeated application of Fig.1 means that the leading pulse, advancing at the full pulse speed $\Delta l/\Delta t$, is being successively halved with each time step, quickly making the leading pulses almost negligible. Within these leading pulses however, the process of slowing down the wave advance is already happening, gradually, and the leading pulses are sending back other pulses to begin increasing the current towards the required value. Thus the "hump" slowly grows, with the ratio of voltage to current rising from 0.5 at the very front towards a value above √_ as the current eventually builds up.

Towards the back of the hump there is "too much" current: the ratio of voltage to current is above the value of $\sqrt{1/2}\, Z_0$ required for continued propagation. So the rear of the hump sees a rise in the voltage ahead due to this extra current, while behind it sees a strongly negative voltage due to the accumulated "borrowing" of current. This sharp voltage gradient causes an abrupt cut-off of current flow, and therefore stops the advance of both voltage and current waveforms.

This process provides another way of seeing how TLM models distributed inductance. At the front of the hump the voltage distribution has a negative spatial gradient, causing a positive rate of change of current, the proportionality between these two being precisely the mesh inductance. As long as this gradient is present, the current will continue to rise in this region. Meanwhile, at the back of the hump, there is a sharp positive voltage gradient that produces a large, negative, time rate of change of current.

The required conditions on p and q in figure 2 (Eqn. 11) are fulfilled "on average" within the hump. The average ratio of voltage to current and therefore the average normalised speed are each exactly $\sqrt{1/2}$. TLM needs a little time and space to achieve this, but once achieved, it maintains it perfectly, indefinitely.

Significantly, for an initial unit voltage impulse ($p = 1$) the sum of the cluster of voltage pulses in the hump oscillates around $\sqrt{1/2}$ while the sum of the corresponding current values remains about 1.0 (normalised), giving the correct effective impedance. Thus the total initial current has been conserved within the hump. Again it is possible to visualise how the current keeps gathering itself up within the cluster until it exhausts the total initial current, and then it abruptly cuts itself off.

The fairly large, quasi-static, quasi-chaotic waves behind the hump provide space for "balancing" all accounts without upsetting the essential characteristics of wave propagation. This is still needed, because although the required special conditions have been achieved in the hump, there are other, apparently contradictory conditions to be fulfilled in the overall progress of the voltage and current distributions. Recall that the entire waveform corresponds to a single unit pulse initially, so for all time and for the entire mesh, there must be

a) conservation of voltage, or $\sum v_i = 1$;
b) conservation of current, or $\sum i_i = 1/Z_0$;
c) conservation of total voltage distribution speed at $(c_v)_x = \Delta l/\Delta t$;
d) conservation of total current distribution speed at $(c_i)_x = {}^1/_2 \Delta l/\Delta t$.

Requirements a) and b) follow form repeated application of figure 1, while c) and d) from repeated application of the definitions.

The rather large area of almost standing waves allows space in which these requirements can be met overall, while allowing the apparently conflicting special requirements to continue in the “hump”. The almost random jumping, of short wavelength over a large area, ensures that even with just a small number of initial pulses in series, the combined effect average to close to zero.

9. Summary and conclusions

A way of looking at 2-D scalar TLM has been presented which is believed to have novel elements. It is based on a “statistical”, time-domain analysis of how voltage and current pulse distributions behave under the action of the basic, simple TLM algorithm of scatter and connect. Pulse distribution speeds for voltage and current pulses have been defined that lead to new ways of arriving at wave speed, wave impedance and other parameters. The approach also provides new “explanations” of a number of sometimes puzzling aspects of TLM usually accounted for only in the frequency or wave domains.

In summary, the single TLM impulse cannot propagate as an advancing impulse without spreading (“dispersing”) a little into a cluster of pulses. This is because the required special conditions for the wave speed and the wave impedance cannot be met in the 2-D scattering of just a single pulse. The spreading continues until the correct average ratio of voltage to current (the mesh impedance) is achieved within this cluster. Then the spreading stops. The speed of advance of this cluster has then become consistent with this impedance. The speed then remains constant. A dynamic balance both keeps the cluster together and controls the speed, indefinitely.

Clearly TLM modelling of waves that are long compared with the cluster size will work very well: but at shorter relative wavelengths the limiting factor is precisely the finite size of the cluster. Thus “wave dispersion” is directly attributable to pulse “dispersion”. Furthermore the pulse dispersion is, and must be, limited over time and in extension if propagation is to be maintained, even at long wavelengths.

This “room to manoeuvre” that TLM needs, to operate correctly, offers a possible explanation why “perfectly matched” boundary layers for modelling open boundaries need a certain size to function properly.

Future work could involve applying the same ideas to lossy TLM and extending the analysis to 3-D and the more complex TLM nodes systems now widely used.