Chapter 3 The theory of TLM: an electromagnetic viewpoint

The previous chapter introduced many of the concepts of TLM from a mechanical viewpoint. However, a large body of literature on the subject approaches TLM from its origins in electromagnetics. This chapter attempts to provide a bridge, so that those who are familiar with mechanical concepts should be able to gain a deeper understanding of the standard theory of the subject. It contains much in common with the equivalent chapter in a related book on TLM modelling of diffusion processes [3.1]. However, in this chapter we will attempt to make fewer assumptions about the level of expertise in electrical network theory. We will cover the basics of both lossless and lossy TLM algorithms. The concepts will initially be treated in terms of lossless processes which can be used to describe a variety of wave propagation phenomena. We will start by introducing

- a variety of relevant electrical components. (resistors, capacitors, inductors etc)
- relevant electrical network theory (Thévenin's theorem)
- We will then discuss mechanical analogues(forces, fields, displacements)

Armed with these we will introduce Maxwell's equations of electromagnetics (only in as much as we need them). We will then consider the behaviour of impulses on a transmission line and at that point we should then have sufficient background to tackle concepts in TLM itself.

The building blocks; electrical components.

Resistor

This is an energy dissipating device which is very similar to a narrow pipe or orifice which controls the flow of a liquid. In the mechanical case we can say that the flow (litres/second) is proportional to the applied pressure (or pressure drop across the device). In electricity the current is the rate of flow of charge and is proportional to the applied voltage. The constant of proportionality is called the resistance and the behaviour can be expressed by Ohm's law ($V = IR$). The magnitude of resistance depends on the resistivity (ρ), a specific property of the conducting material and on its geometry. Thus the resistance of a conductor of length (L) and cross-sectional area (A) is given by

$$R = \rho\, L/A \tag{3.1}$$

The resistivity depends on the physical processes in the conductor, namely the concentration of charge conducting species and their mobility (a measure of the achievable drift in a unit electric field).

The power (Joules/second) which is dissipated in a resistor can be expressed in three possible ways (IV, I^2R or V^2/R)

Capacitor

A capacitor is an energy storage device. The entity which is stored is electrical charge and for a given geometry there is a relationship between the charge (Q measured in Coulombs) and the voltage ($Q = CV$) where the constant of proportionality (C) is called the capacitance and has units (Farads). One of the simplest geometries is an arrangement of two parallel plates of area (A), separated by a distance (L) in vacuum. The capacitance is then expressed as

$$C_0 = \varepsilon_0 A/L \tag{3.2}$$

ε_0 is called the permittivity of free-space and has a value 8.854×10^{-12} F m^{-1}.

If the medium between the two plates is not a vacuum, then it will influence the charge storage capacity (in general it will be possible to store more charge) and the extent is expressed in the 'relative permittivity' (ε_r). Thus the capacitance between two plates in some general medium can be written as $C = \varepsilon_r C_0$.

As a general guide to magnitude, we can say that two tailoring pins with 2mm diameter heads, separated by the width of an average human hair (25µm) have a capacitance of 1.11pF (1pF, pico Farad = 10^{-12} Farad).

A capacitor has an additional effect that must be considered when it is used in a circuit where the applied voltage is a function of time. It introduces a phase delay between applied voltage and the flow of charge (current). This can be best demonstrated for an alternating (AC) signal of constant frequency and is shown in figure 3.1

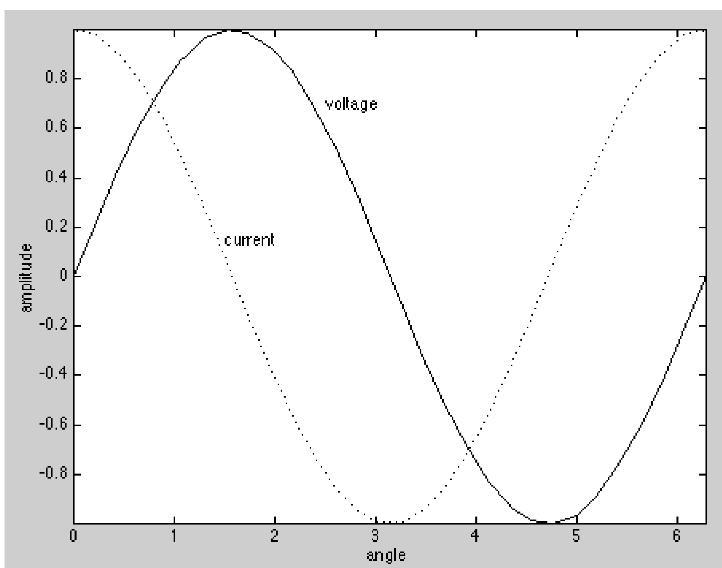


Figure 3.1 phase-lag between AC voltage and current in a capacitor

It is clear that the phase angle is $\pi/2$ and this is normally expressed as a function of frequency by representing the AC equivalent of Ohm's law using the imaginary number, j ($\sqrt{-1}$):

$V = I Z$

where $Z = 1/(j\omega C)$ is called the 'impedance'. Circuits which involve connections of resistors and capacitors can have an impedance which is represented using complex vectors as $Z = A - jB$. Impedances in parallel or series are added exactly as if they were resistors (i.e. given Z_1 and Z_2 in series the sum is $Z_1 + Z_2$, in parallel the sum is $[1/Z_1 + 1/Z_2]^{-1}$).

The capacitor can be viewed as an energy storage device and the magnitude of the energy stored in the electric field between the conducting surfaces of the capacitor is given by $CV^2/2$.

Series connections of resistors and capacitors introduce voltage-current phase delays that are different from $\pi/2$. If a step-function change in voltage is applied to such a circuit then a voltage transient is observed across the capacitor. This is given by:

$$V(t) = V_\infty(1 - \exp[-t/RC]) \qquad (3.3)$$

(V_∞ is the fully charged voltage, RC is the circuit time-constant)

If the ends of the circuit are subsequently connected so that the capacitor discharges through the resistor, then the voltage at any subsequent time is given by

$$V(t) = V_\infty \exp[-t/RC] \qquad (3.4)$$

RC charge and discharge have very close analogues in mechanical engineering.

Inductor

An inductor is a device where the stored entity is magnetic field. If an attempt is made to change the current through a coil which is maintaining this field then there will be a voltage established which tries to prevent this change. This is given by Lenz's law:

$$V = -L(dI/dt) \qquad (3.5)$$

dI/dt is the rate of change of current, and the constant of proportionality (L) is called the inductance and has units, Henries. If the medium which is surrounded by a current carrying coil is ferromagnetic rather than air or vacuum, then the inductor can store a significantly larger magnetic field. The inductance (L) is related to L_0 through the relative permeability, μ_r, which for the case of ferromagnetic materials can be 10,000 or more.

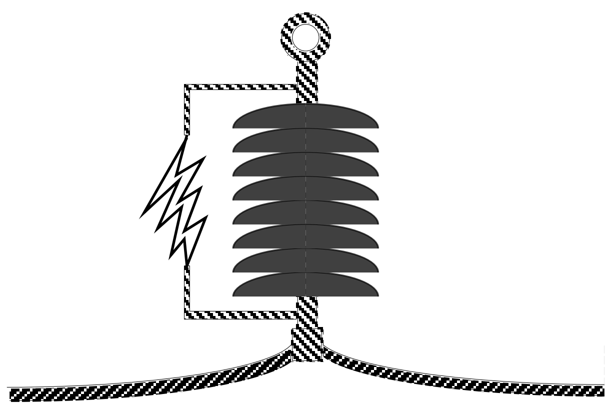


Figure 3.2 Insulator necklace on a high voltage (on-pylon) support showing a flash-over.

As a measure of inductance we can consider a length of overhead high-voltage line which has an effective inductance of 1mH. If this is hit by a lightning stroke so that 1,000A enters the line during 1μs then dI/dt = $10^9 As^{-1}$. The induced voltage given by L(dI/dt) will be 10^6V which exceeds the withstand capability of the line and must be dissipated in a flashover (see figure 3.2).

An inductor also introduces a 90º phase delay between applied voltage and current which is shown in 3.3.

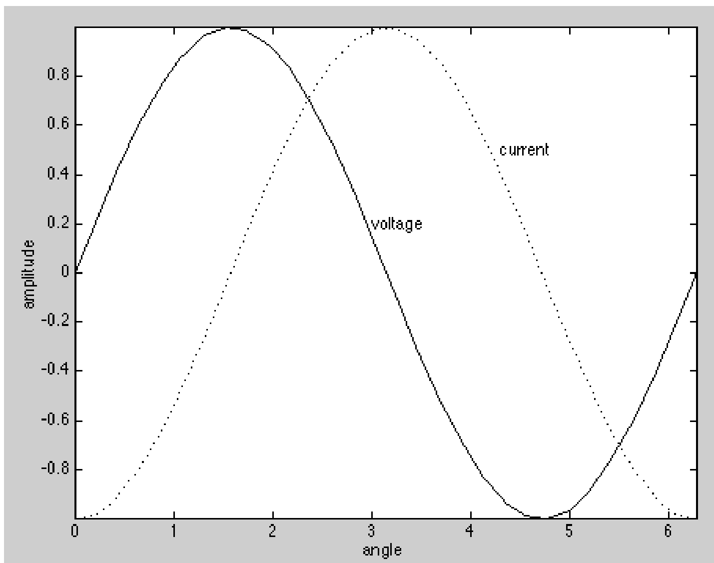


Figure 3.3 phase-lag between AC voltage and current in an inductor

This relationship is normally expressed by another representation of Ohm's law

$$V = I Z \quad (3.6)$$

where Z = jωL

Circuits which involve connections of resistors and inductors can have an impedance which is represented using complex vectors as Z = A + jB. Connections of inductors and capacitors have a particularly interesting property, they resonate. This is because the capacitor stores electrical energy while the inductor stores magnetic energy, and the system oscillates as energy continually changes from one form to the other. This is exactly analogous to mechanically oscillating systems such as a pendulum or a spring and mass, where the energy alternates between potential and kinetic.

Transmission line

The transmission line an arrangement of conductors to guide electromagnetic energy flow. At its simplest it consists of a pair of wires, or even a single wire close to a ground plane. It has inductance and capacitance distributed along its length. It can be represented as a continuous distribution of series inductors and parallel capacitors which act as shunts to ground. We could measure the capacitance of a length of line and dividing by the length we would get a value C_d, the distributed capacitance per meter. We could make a similar measurement of inductance per unit length to arrive at L_d. Although these are continuous properties we will frequently treat them as discrete parameters which are lumped together within regions of space and separated from each other by ideal conductors.

Electromagnetic theory which is outside the scope of this book can be used to predict the impedance of a length of lossless transmission line. This is given by

$$Z = \sqrt{\frac{L_d}{C_d}} \tag{3.7}$$

But since this is the same as $\sqrt{\frac{L}{length}\frac{length}{C}}$ the impedance is independent of the length of the transmission line.

If a signal is travelling along a lossless transmission line whose impedance is Z_0 (called the characteristic impedance) then it will continue undisturbed until it encounters a discontinuity.

This may be the end of the line (open-circuit termination), a short-circuit to ground or some transmission line with another impedance. If we assume that the termination has an impedance Z_T then a portion of the signal (depending on the magnitude of Z_T) will be reflected back on itself. The reflection coefficient (ρ) is then given by the equation:

$$\rho = \frac{Z_T - Z_0}{Z_T + Z_0} \qquad (3.8)$$

Z_T might be ∞ (an open-circuit) so that ρ = 1. Z_T might be a short-circuit so that ρ = -1. We could consider a TV aerial which has a 50Ω coaxial down-feed cable. The naïve user might decide to connect two TV sets to this without any matching circuit. In this case Z_T = 25Ω so that ρ = -1/3. This effectively means that each TV set obtains 4/9 of the total power, which in weak reception areas may not be satisfactory.

Transmission lines have one important effect which is central to the concept of TLM modelling; they introduce a time-delay. The capacitance and inductance contain parameters ε_0 and μ_0 and the product $1/(\varepsilon_0 \mu_0)$ is equal to the square of the speed of light in a vacuum. The medium of a transmission line through which an electromagnetic signal travels has permittivity $\varepsilon_r\varepsilon_0$ and permeability $\mu_r\mu_0$ so that the velocity of propagation is reduced. Transmission lines can be specially constructed to act as delay-lines and these have a wide range of applications.

Basic network theory

Will assume Ohm's law which in any event has close analogues in hydraulics (flow is proportional to pressure drop). We are particularly concerned with Thévenin's theorem, which leads to a very useful approach to analysing the behaviour of an electrical circuit. The theorem says that any circuit where a measurement is made at a pair of terminals can be replaced by

- a voltage source which is equivalent to the voltage that would be observed if there was no external connection to the terminals (*open-circuit voltage*)
- an impedance which is equivalent to all the impedances of the circuit if all of the voltage sources had been replaced by perfect conductors (*short-circuit impedance*)

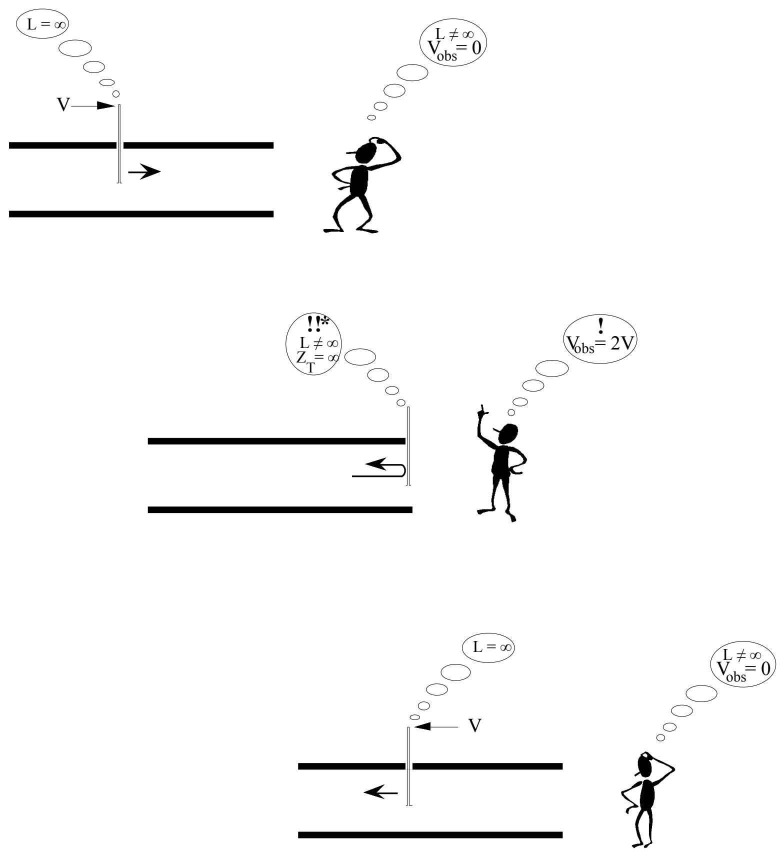


Figure 3.4 Time-history of an impulse travelling on a transmission line from two points of observation

If we take the transmission line and look at the time history of an impulse travelling along it as observed from one end, we will see how Thévenin's theorem is applied in TLM. In the first picture in figure 3.4 we see a pulse travelling on what it believes is an infinite transmission line. The outside observer is aware that it is not infinite but can see no signal. In the second picture the pulse suddenly becomes aware that it is faced with an open circuit termination and according to eqn. (3.6) the reflection coefficient, $\rho = 1$. At that instant the voltage which is seen by the outside observer is the sum of the incident and reflected pulses, i.e. 2V. Thereafter the observer sees nothing and once again, the reflected pulse has no knowledge of the finite nature of its environment.

In compliance with the second of the Thévenin requirements we could take the same line in the absence of any pulse and short the other end. An impedance measurement would yield the value, Z. The Thévenin equivalent circuit of a transmission line is shown in figure 3.5.

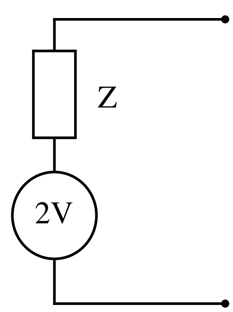


Figure 3.5 Equivalent circuit of a transmission line <u>at the moment of arrival</u> of a signal.

The Thévenin equivalent for a transmission line and figure 3.4 indicate some of the significance of TLM. Because the impulse travelling on the line is unaware of anything except its immediate surrounds we are able to treat each impulse independently of any other. Any interaction occurs only when they meet. This effectively time discretises the problem, and further, the smallest time unit that can be considered is the interval between arrival of impulses at observation points. This value Δt is normally chosen so that the velocity along a length of line Δx can be represented as

$$V = \frac{\Delta x}{\Delta t} \tag{3.9}$$

So, once we have constructed the correct analogue, our algorithms become little more than an efficient method to keep track of impulses. We can also approximate more complicated waveforms by means of a train of impulses as shown in figure 3.6.

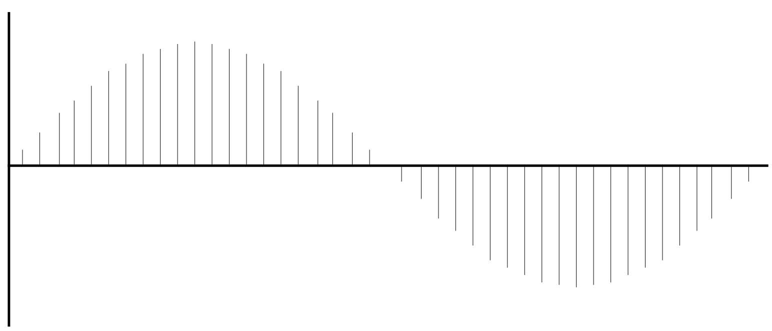


Figure 3.6 A discrete sinusoid constructed from a time series of impulses

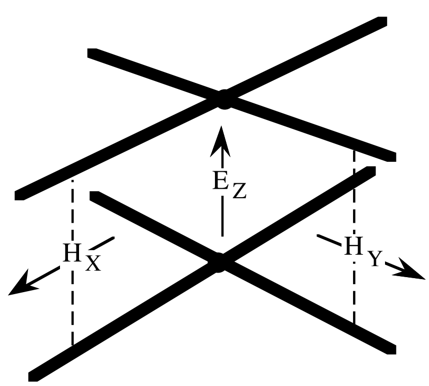


Figure 3.7 intersecting pair of two-wire transmission lines

Propagation of a signal in space (Maxwell's equations)

The behaviour of these impulses can be related to the propagation of electromagnetic waves and an analysis leads to a wave equation. Different network formulations lead to expressions for different field components. If an assembly of transmission lines is connected as shown in figure 3.7 then we have what is called a 'shunt' transmission-line network. If the network had resistive losses then the electric field vector, E_z would be expressed by the lossy wave equation.

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} + \varepsilon\sigma \frac{\partial E_z}{\partial t} \qquad (3.10)$$

The first term on the right expresses a wave propagation with a velocity $\mu\varepsilon = 1/c^2$. The second term, which contains $\varepsilon\sigma$, expresses an attenuation of the wave. If the transmission line is lossless then $\sigma = 0$ and the more familiar wave equation is obtained:

$$\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} = \mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} \qquad (3.11)$$

There are similar expressions for all of the other field components and for a large part of this book eqns (3.10) and (3.11) represent as much knowledge of Maxwell's equations as is required.

Distributed and lumped circuits

Up to now the concept of a transmission line has really been that of a distributed system. There was inductance distributed along the conductor. Resistance, if present, was also along the conductor. Capacitance per unit length was distributed between conductors and if there was leakage between conductors then this was also distributed. It is quite difficult to deal with such systems, and we try where possible to replace them by an equivalent with lumped components. In fact, we have already done this in figure 3.5 where the parameters of a transmission line are replaced by their Thévenin equivalents. Now, everything that has been presented above about Maxwell's equations can also be expressed for a lumped component electrical network consisting of resistors, capacitors and inductors. In this case we will use analogues, where potential, V replaces the electric field, E and current, I ≡ replaces the magnetic field, H. We will start by considering a one-dimensional case (figure 3.8) which has lumped components whose values are equivalent to what are called *distributed* parameters. Thus, R_d is the resistance per unit length = R/Δx. C_d and L_d are similarly defined.

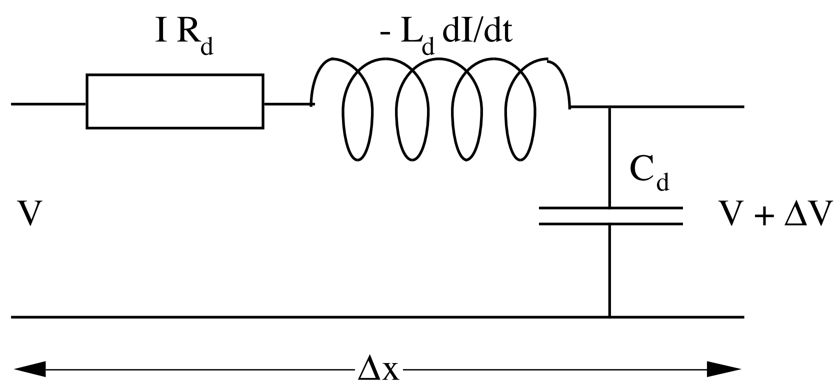


Figure 3.8 A simple LRC circuit

The change in voltage as a function of distance depends on the voltage drop across the resistor (IR) and the voltage which develops across the inductor (-LdI/dt).

$$\text{Thus} \quad \lim_{\Delta x \to 0} \frac{V + \Delta V - V}{\Delta x} = \frac{\partial V}{\partial x} = -IR_d - L_d \frac{\partial I}{\partial t} \qquad (3.12)$$

At the same time there is a change in current over this distance due to the change in charge being stored on the capacitor.

$$I = C\frac{\partial V}{\partial t} = C_d \Delta x \frac{\partial V}{\partial t} \text{ so that}$$

$$\lim \Delta x \to 0 \quad \frac{I + \Delta I - I}{\Delta x} = \frac{\partial I}{\partial x} = -C_d \frac{\partial V}{\partial t} \tag{3.13}$$

Now, if eqn (3.12) is differentiated with respect to x then we get

$$\frac{\partial^2 V}{\partial x^2} = -R_d \frac{\partial I}{\partial x} - L_d \frac{\partial I^2}{\partial x \partial t} \tag{3.14}$$

Eqn (3.13) and its derivative can now be substituted into this to give:

$$\frac{\partial^2 V}{\partial x^2} = L_d C_d \frac{\partial V^2}{\partial t^2} + R_d C_d \frac{\partial V}{\partial t} \tag{3.15}$$

This is called the *Telegraphers'* equation and is the basis for the TLM method. The extension of this equation to two and three-dimensions will be deferred until we consider specific TLM networks. However, before we proceed further it is necessary to revisit some additional basic electromagnetic theory relating to transmission lines.

Transmission lines revisited

Time discretisation

The concept of distributed parameters which was mentioned is central to our model for a transmission line. Eqn (3.7) represented the impedance of a lossless transmission line, regardless of length.

the velocity of an impulse on a transmission line can be given by:

$$v = \sqrt{\frac{1}{L_d C_d}} \qquad (3.16)$$

Much of the analysis of the behaviour of electromagnetic fields is presented in the complex domain [3.2] assuming harmonic (sinusoidal) excitation, so that variables of a given frequency are characterised by a magnitude and a phase. A propagation constant γ can then be defined so that a wave can be presented as $A\sin(\gamma x)$. The propagation constant of an impulse on a line is then given by:

$$\gamma = j\omega\sqrt{L_d L_d} \qquad (3.17)$$

The time taken for an electrical signal impressed on the line to traverse it will be determined by the local velocity of light, which depends on $\mu\varepsilon$ as mentioned above.

If the resistance on a line is such that R_d is non-negligible then the situation is much more complicated

$$Z_0 = \sqrt{\frac{R + j\omega L}{j\omega C}} \qquad (3.18)$$

$$\gamma = \sqrt{(R + j\omega L)\, j\omega C} \qquad (3.19)$$

For many years the textbook approaches to electromagnetics have explored ways of circumventing this problem. If R_d is negligible then the Telegraphers' equation reduces to a simple wave equation and this is the basis for lossless TLM modelling. If, on the other hand R_d is significant then there is an entire branch of TLM modelling which seeks to ignore the wave component in eqn (3.15) so that it can be treated as a diffusion equation [3.3, 3.4].

The velocity of propagation on a uniform, lossless transmission line can be related to its parameters:

$$v = \frac{\Delta x}{\Delta t} = \sqrt{\frac{1}{L_d C_d}} = \sqrt{\frac{\Delta x^2}{L\,C}} = \Delta x \sqrt{\frac{1}{L\,C}} \qquad (3.20)$$

or
$$\frac{1}{\Delta t} = \frac{1}{\sqrt{L\,C}} \qquad (3.21)$$

now, using $Z = \sqrt{\frac{L}{C}}$ we can eliminate either L or C in the eqn (3.21) to get

$$Z = \frac{\Delta t}{C} \quad \text{or} \quad \frac{\Delta t}{C_d \Delta x} \qquad (3.22)$$

$$Z = \frac{L}{\Delta t} \quad \text{or} \quad \frac{L_d \Delta x}{\Delta t} \qquad (3.23)$$

These relationships between the line impedance, the line parameters and the spatial and temporal discretisations are the fundamental building blocks of TLM.

Discontinuities

Eqn. (3.8) expresses the behaviour of a pulse/wave on a line when it encounters a change of impedance. The signal which is transmitted past a discontinuity in a line depends on whether we are dealing with current or voltage in our analysis. Both the current and voltage expressions for transmission and reflection are essential in the development of lossless and lossy TLM algorithms and these will be treated in turn. If we are dealing with current (charge per second) then the conservation of charge controls what happens:

$$ {}^{i}I = {}^{r}I + {}^{t}I \tag{3.24} $$

in this case the super-scripts i, r and t indicate whether the current is *incident, reflected* or *transmitted*. The reflected current can be defined using the reflection coefficient as:

$$ {}^{r}I = \rho \, {}^{i}I \tag{3.25} $$

therefore the transmitted current is:

$$ {}^{t}I = (1-\rho) \, {}^{i}I \tag{3.26} $$

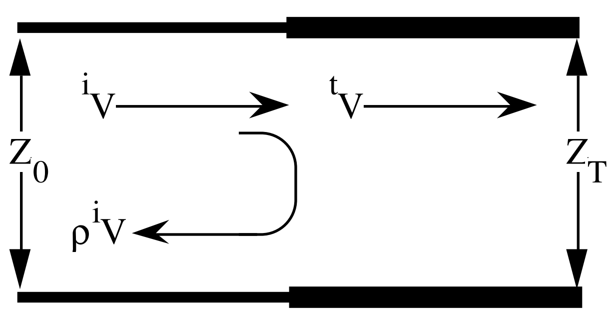


Figure 3.9 Voltage scattering at an impedance discontinuity

An analysis of the scattering of voltage pulses (figure 3.9) starts from the same position but recalls that voltage is a measure of work done when charge is moved against an electric field. Equation (3.24) can be restated using Ohm's law as:

$$\frac{{}^{i}V}{Z_0} = \rho\frac{{}^{i}V}{Z_0} + \frac{{}^{t}V}{Z_T} \tag{3.27}$$

However, by virtue of eqn (3.8) we can write this in terms of ${}^{t}V$ as:

$${}^{t}V = (1-\rho){}^{i}V\frac{Z_T}{Z_0} = (1+\rho){}^{i}V \tag{3.28}$$

It is interesting to note that eqns (3.26) and (3.28) can be combined to given the transmitted power, P:

$${}^{t}P = (1-\rho){}^{i}I(1+\rho){}^{i}V = (1-\rho^2){}^{i}P \tag{3.29}$$

Since the reflected power is $\rho^2\,{}^{i}P$ this confirms the energy conservation law.

TLM nodal configurations

Conventional TLM has commonly used what is called the *shunt* node configuration. This assumes that current is equivalent to magnetic field and voltage is equivalent to electric field. In a two dimensional problem I_x and I_y are the analogue of H_x and H_y, and V_z is equivalent to E_z as can be seen in figure 3.7. There is another approach called the *series* node configuration which uses E_x, E_y (equivalent to V_x, V_y) and H_z equivalent to I_z. Details of the implementation of this and the more complicated three-dimensional lossless representations can be found in references such as Christopoulos [3.5]. We will concentrate here on the development of lossless TLM algorithms based on the shunt node in one- and two-dimensions.

The conversion of these ideas into practical TLM algorithms requires the use of two simple assumptions, one equation from electromagnetics and one theorem from basic electrical theory, all of which has been covered earlier

- The first assumption is that all data (field amplitudes) are represented by impulses of very short duration. Thus an impulse entering a transmission line has no knowledge about the length of the line; indeed it is unaware that the line is finite until it arrives at a discontinuity (see figure 3.4).

- The second assumption is needed for computational purposes and requires that all pulses moving around the spatial mesh are synchronised, in the sense that they all arrive at the next nodal intersection or discontinuity at the same instant.

- The third assumption is that the reflection at an impedance discontinuity given by eqn (3.8) applies to the pulses considered here .

We are now in a position to monitor the progress of a single pulse as it starts to scatter around the network of nodes, or mesh which describes our problem. For our purposes we will consider a two-dimensional system and since we are dealing with a Cartesian coordinates, we can describe the directions of scatter by the compass directions N, S, E and W. Let us consider ${}^{i}V_{W}$ which is incident from the west onto a two-dimensional TLM node (x,y) as shown on the left in figure 3.10.

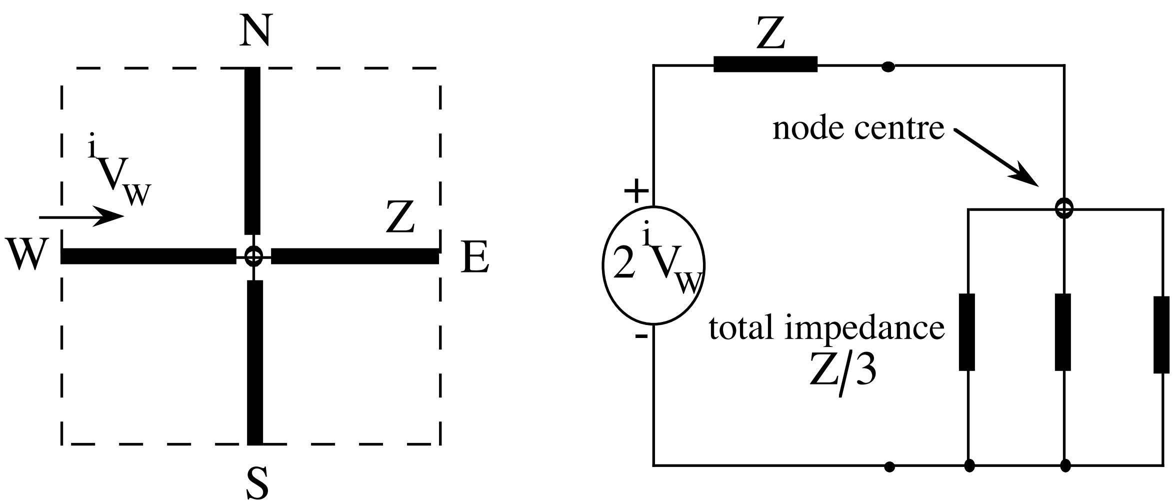


Figure 3.10 Lossless TLM node and its Thévenin equivalent

As the pulse (from the west) arrives at the end of its line it sees what it believes are three lines of apparently infinite length (representing north, south and east) connected in parallel and representing a terminating impedance of as shown on the right of figure 3.10.

Impedances add in parallel as $\frac{1}{Z_T} = \frac{1}{Z} + \frac{1}{Z} + \frac{1}{Z}$ so that $Z_T = Z/3$.

The pulse undergoes scattering according to eqn (3.8) and since $Z_T = Z/3$ we have $\rho = -1/2$. This means that a pulse of magnitude $(-0.5)^{i}V_W$ is returned down the incoming transmission line. The remainder of the signal is transmitted into the other arms. Pulses incident from arms N, S and E are simultaneously incident and undergo scattering. They also contribute to the voltage at the node centre which can be represented by the principle of superposition. This states that the voltage at a node is the sum of the currents divided by the sum of the admittances (where admittance is reciprocal of impedance):

$$_k\phi(x,y) = \frac{\sum I}{\sum Y} = \frac{\left[\frac{2^iV_N}{Z} + \frac{2^iV_S}{Z} + \frac{2^iV_E}{Z} + \frac{2^iV_W}{Z}\right]}{\left[\frac{1}{Z} + \frac{1}{Z} + \frac{1}{Z} + \frac{1}{Z}\right]}$$

or

$$_k\phi(x,y) = \frac{\left[{}^iV_N + {}^iV_S + {}^iV_E + {}^iV_W\right]}{2} \qquad (3.30a)$$

In a situation where an n-dimensional network has 2n arms with different impedance then eqn (3.30a) can be expressed in a general form as:

$$_k\phi(x,y) = \frac{\sum_{j=1}^{n} I_j}{\sum_{j=1}^{n} Y_j} = \frac{\left[\frac{2^iV_1}{Z_1} + \frac{2^iV_2}{Z_2} + \frac{2^iV_2}{Z_3} + \ldots\ldots\right]}{\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots\ldots\right]} \qquad (3.30b)$$

The pulse which is scattered back in any direction is the sum of what is reflected and what is transmitted from all other arms. Thus

$$^sV_W = \rho\,{}^iV_W + \tau({}^iV_N + {}^iV_S + {}^iV_E) \qquad (3.31)$$

There are similar equations for ${}^iV_N$, ${}^iV_S$, ${}^iV_E$ and ${}^iV_W$ and the entire scattering process can be expressed in matrix form as:

$$_k\begin{pmatrix} {}^sV_N \\ {}^sV_S \\ {}^sV_E \\ {}^sV_W \end{pmatrix} = \mathbb{S}\ {}_k\begin{pmatrix} {}^iV_N \\ {}^iV_S \\ {}^iV_E \\ {}^iV_W \end{pmatrix} \qquad (3.32)$$

where $\mathbb{S}$ is the scattering matrix and is given by

$$\mathbb{S} = \begin{pmatrix} \rho & \tau & \tau & \tau \\ \tau & \rho & \tau & \tau \\ \tau & \tau & \rho & \tau \\ \tau & \tau & \tau & \rho \end{pmatrix} \tag{3.33}$$

In this general form it can be applied to any type of two-dimensional TLM (lossless or lossy) scattering. The situation where impedances are equal and do not contain any resistance (as shown in figure 2.9) has $\rho = -0.5$ and $\tau = 0.5$. The scattering matrix for lossless TLM is then

$$\mathbb{S} = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \tag{3.34}$$

Each pulse travels the discretisation distance Δx during the discretisation time Δt after which it becomes an incident pulse at an adjacent node. The connections to other nodes as seen at node (x,y) can be expressed in terms of space and time-step, k+1 as

$$_{k+1}{}^{i}V_N(x,y) = {}_{k}{}^{s}V_S(x,y+1) \tag{3.35}$$

$$_{k+1}{}^{i}V_S(x,y) = {}_{k}{}^{s}V_N(x,y-1)$$

$$_{k+1}{}^{i}V_E(x,y) = {}_{k}{}^{s}V_W(x+1,y)$$

$$_{k+1}{}^{i}V_W(x,y) = {}_{k}{}^{s}V_E(x-1,y)$$

The repeated application of the processes of *Scatter* (eqn. (3.32)), *Connect* (eqn. (3.35)) and *Summation* (eqn. (3.30)) for every time step constitutes the entire TLM process for a two-dimensional transmission line network. Before proceeding to the next section, which considers what happens when impulses on TL networks interact with boundaries, we should remind ourselves that lossless formulations that use the scattering matrix as shown in eqn. (3.34) can be used to model wave propagation problems. The inclusion of resistive losses within such networks yields reflection coefficients which are different from -0.5. In these situations which can give good approximations to diffusion processes eqn. (3.33) must be used for the scattering matrix.

Boundaries

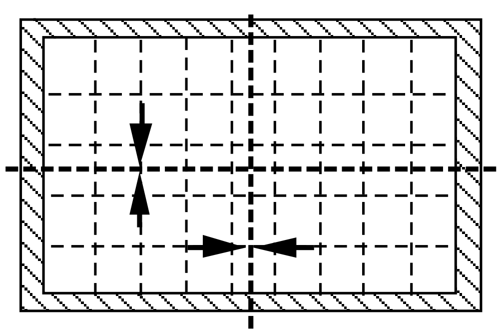
Any physical problem has boundaries and we next start to consider how these might be treated in TLM. Because the technique has evolved from electromagnetics and particularly from microwave theory it defines boundaries in these terms, but there are equally good mechanical definitions (e.g. in acoustics boundaries with specified acoustic pressures or velocities)

- $Z_T = \infty$ corresponds to an electrical open-circuit termination. This means that a voltage (or electric field) pulse which arrive at a boundary are reflected in-phase, because $\rho = 1$. The current at the boundary is zero. This is sometimes called a 'mirror' boundary. In acoustics where we frequently use voltage as the analogue for pressure and current as the analogue for velocity, this corresponds to a rigid boundary, where the velocity into the boundary is zero, and the associated pressure is typically a maximum.

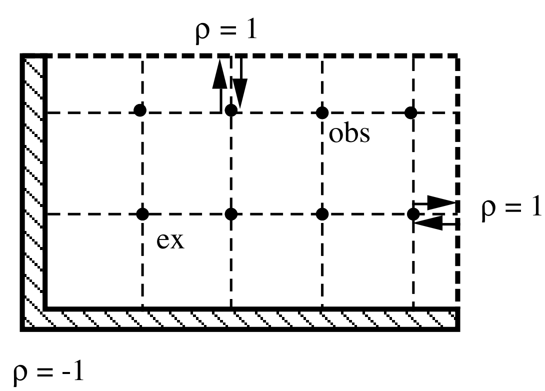
- $Z_T = 0$ is equivalent to an electric short-circuit termination. $\rho = -1$ and any pulse incident on a boundary will be reflected in anti-phase. The voltage (electric field) at such a boundary is zero, and typically the current is large. In acoustics, the acoustic velocity is a maximum and the acoustic pressure is zero, which accounts for the description 'pressure-release' boundary.

- The situation where $Z_T = Z$ is called a 'matched load' boundary condition and $\rho = 0$. The use of this definition requires caution because Z_T is generally a function of frequency. If $\rho = 0$ at one particular frequency, then it may be different from zero at all other frequencies. The definition of a broad-band Perfectly Matched Load (PML) boundary will be discussed later.

The concept of open-circuit boundaries can also be used to reduce the size of the problem which needs to be computed. Frequently an axis of symmetry corresponds to a line along which there is zero current, which is equivalent to a perfectly reflecting boundary, with $\rho = 1$. Consequently, in the case of the rectangular wave-guide that was analysed by Johns and Beurle [3.6], the entire cross-section of the waveguide could be analysed by having two short-circuit boundaries (the outer walls and two open circuit boundaries, the two orthogonal symmetry axes (see figure 3.11).



(a)



(b)

Figure 3.11 (a) A section of discretised electromagnetic waveguide showing the passage of impulses (arrows) across the two orthogonal symmetry axes. (b) Symmetry-reduced model. ('ex' is the point at which excitation signals are inserted into the mesh, 'obs' is the observation point where data is collected for subsequent use in Fourier analysis).

Conclusion

This chapter has demonstrated the basic simplicity of the algorithms of TLM. Much of the perceived difficulty with the technique lies in the range of concepts which together constitute the underlying physics. There must be some level of understanding of these if TLM is to be anything other than the repeated application of a set of rules in the manner of a cellular automaton. We could reasonably say that TLM is an example of a physical interpretation of certain CA rules.

Much TLM research is concerned with the analysis of physical problems and their representation as parameters within a lossless or lossy wave equation. The next chapter develops many of the ideas that were introduced here and applies them to problems in acoustic propagation.

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