TLM modelling of devices with acoustic-mechanical coupling

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ABSTRACT

TLM is here applied to modelling air-borne acoustic waves, to modelling mechanical waves in membranes, and to the coupling between these two dynamic systems. This mechanical-acoustic coupling is crucial to many acoustic devices such as microphones, loudspeakers, headphones and musical instruments. The model is verified using analytically solvable problems.

1. Acoustic TLM: some general comments

TLM models the dynamic interaction of two variables (such as voltage and current, electric and magnetic fields, acoustic pressure and velocity, stress and strain rate), in which the spatial rate of change of one variable causes a time rate of change of the other, and vice versa. This reciprocal interaction leads to the propagation of waves. The product of the two variables has the dimensions of power, while the ratio has that of impedance.

In the node scattering process at the heart of TLM, the value of one of the two variables is caused to be common to all the branches at the node. Because it cannot have several branch directions at once, this variable should therefore be a *scalar* quantity. It is usually associated with a physical quantity characterised by a "potential" variable (or an "effort" or "forcing" or "intensity" or "across" type of variable). The second variable will be of the "flow" or "through" or "flux" type, which can indeed correspond to a vector quantity, the direction of which is modelled by the branch orientation. The TLM scattering algorithim conserves the total "flow" of this second variable, and this requirement, combined with the requirement of the common "effort" variable, gives the scattering matrix.

Looked at from this perspective, acoustic waves are inherently suited to being modelled by TLM in a way in which electromagnetic waves are not. This is because in acoustics, the "effort" variable, acoustic pressure, is indeed a scalar quantity, while the "flow" variable, acoustic velocity, is a vector quantity. One consequence is that acoustic TLM extends easily and naturally from 2-D to 3-D. This is by contrast with electromagnetic TLM, where both of the interacting variables (electric and magnetic fields) are vectors and, precisely because of this, 2-D modelling is relatively simple (where one of the variables can be represented as a scalar) while 3-D is considerably more complex.

In acoustic TLM ideal acoustic pressure wave fronts (or step pulses) are assumed to propagate along ideal acoustic transmission lines (narrow tubes with hard internal surfaces). The wave fronts travel at the acoustic wave speed, changing the fluid velocity as they pass [1]. The ratio of the magnitude of an acoustic pressure pulse, to the acoustic velocity it produces, is the acoustic impedance $Z = \rho c$, where ρ is the density and c is the wave velocity. During each time increment two pressure pulses, p^+ and p^-, travel in opposite directions in each branch. The acoustic pressure in the branch is taken as the sum of these two pulses, $(p^+ + p^-)$, while the associated acoustic velocity, in the p^+ direction in the branch, will be $(p^+ - p^-)/Z$.

2. Acoustic Boundaries

"Passive" acoustic boundaries of different kinds are easily modelled. Boundaries can be massive and rigid, giving total reflection of pressure pulses and therefore zero normal velocity; or, perfectly yielding, giving total reflection with inversion and so zero acoustic pressure; or, absorbing with partial reflection. Open space boundaries (that is boundaries at infinity, or totally absorbing boundaries) are not as simple and, in this study, were modelled by a mesh region of successively greater absorption, with an attenuation factor in the TLM algorithm ranging from near unity at the boundary to near zero at the end of the absorbing region (say 10 nodes deep).

The main interest in the present paper was the study of acoustically active boundaries, and the associated mechanical-acoustic interaction, such as found in loudspeakers, microphones, musical instruments and similar acoustic sources or sensors. In this case the mechanical system boundary can both reflect and source/sink acoustic energy, and this coupling of two vibrating systems is crucial to the operation of such devices, and therefore to their modelling.

2.1 Light but rigid vibrating boundary

Firstly, a rigid (that is, non-flexing) but acoustically light (low inertia) vibrating boundary was considered. To verify the TLM modelling approach it was applied to the well-known case of a second order mechanical driver piston (mass, spring, damper) exciting one end of an acoustic tube closed at the opposite end: Fig.1.

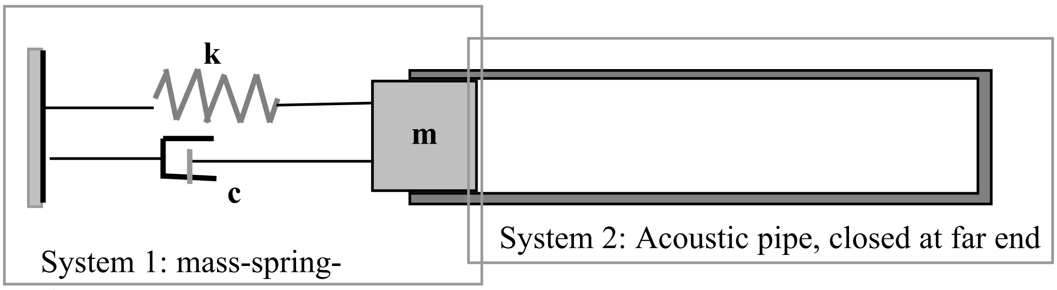


Fig.1: The driver pipe system: a simple coupling of two systems

The amplitude of driver vibration will be small in comparison with the TLM mesh spacing. There is a two-way interaction. Firstly, the coupling from the air acoustics ("System 2") to the driver was modelled by considering the instantaneous acoustic pressure as an external driving force acting on the driver's inherent dynamics (second order mechanical system, "System 1"). The second coupling, from the driver to the air acoustics, was achieved by modifying the rigid boundary reflection condition so that the reflected pressure pulse was changed by an amount $\Delta p = \rho c v$, where ρc and is the acoustic impedance and v is the instantaneous actuator boundary velocity, taken as positive if the boundary is moving towards the field.

The frequency response can be obtained indirectly from the Fourier transform of an impulse response of the system. Thus the driver was given an initial velocity impulse, and the output recorded. The chosen "output" of interest was the pressure at the closed end of the tube.

This problem has been analysed analytically [3] by matching of the multiple resonance frequencies in the tube with the driver resonance frequency, the matching boundary condition being the common acoustic and driver velocity on the face of the driver. Between the two systems, the tube resonances will dominate if the driver mass is small, whereas the driver resonances will tend to dominate if its inertia is large compared with the tube's acoustic mass. In both cases however each subsystem modifies the resonance frequencies of the other, as expected. In these and similar TLM model runs, excellent agreement was found between the numerical TLM results and the analytically expected results, such as given by Kinsler et al [3].

2.2 Flexible membrane boundary

An example of more complex mechanical acoustic interaction is when the active boundary is a thin membrane under tension (as found for example in a condenser microphone) with its own inherent multiple modes of vibration and characteristic frequencies. Such a membrane, vibrating in a vacuum, obeys the 2-D wave equation, so it was decided to adapt the TLM to model this system, and then have the two TLM systems (membrane and air-borne acoustic) interacting with each other to simulate the membrane-air interaction.

The TLM model of a circular membrane in vacuum (no external damping) was developed first. Regarding the basic variable in the wave equation, rather than choose the "obvious" local membrane normal displacement, z, from the neutral position, it was found better to use a $T(\delta z/\delta x + \delta z/\delta y)$ type term as the "effort" variable (analogous to acoustic pressure or voltage) and $\delta z/\delta t$ as the "flow" variable (analogous to acoustic velocity or current), where T is in N/m, the membrane tension per meter width.

Firstly, the vacuum membrane model was tested, both in free vibration (by giving it an initial displacement and/or velocity) and in forced vibration (point driving force and uniformly distributed driving force). The expected behaviour was observed, with the various lower-order modes of vibration easily reproduced, either by appropriate initial conditions or by appropriate driving frequencies at suitable driving locations. Because the primary TLM variable was the normal force (T $\delta z/\delta x$ + T $\delta z/\delta y$), external driving consisted simply of adding (or subtracting) the external force (pressure by area) to the TLM pulses at each node, and letting them propagate in the normal TLM (wave equation) way.

The coupling of the membrane TLM with the acoustic TLM was now relatively straightforward. Local membrane velocity, v = dz/dt, caused the injection of acoustic pressure pulses of magnitude $\rho c v$ into the acoustic TLM, while the local acoustic pressure became a force input to the membrane TLM system. The distinct wave velocities in the air and in the membrane differed by a factor of about 1.4, so to keep the two systems synchronised (same time increment Δt), the mesh spacing (Δl) in the two meshes were made to differ by this ratio.

3. Simplified condenser microphone model

Finally to test the combination, it was decided to model a condenser microphone seen in Fig. 2, to obtain its frequency response. The incoming plane wave was assumed to approach the microphone normally and impinge on one side of the membrane, behind which was the electrode and an acoustic chamber with air holes. The attenuation due to viscous damping associated with air movement behind the membrane was in this work modelled simply by a viscous damping term added to the membrane TLM system.

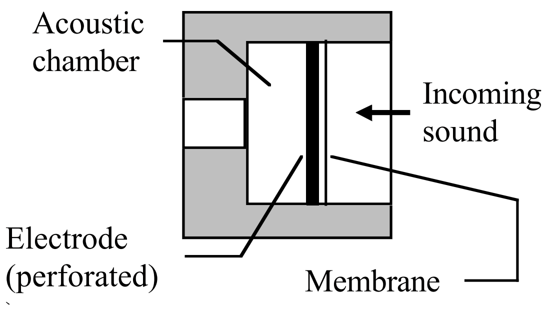
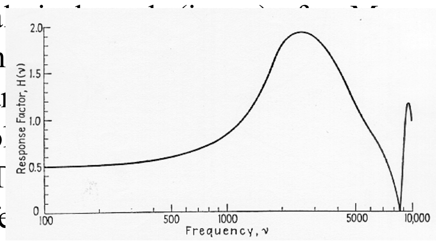


Fig. 2. A simple condenser microphone model as simulated

Although both the membrane and the air TLM models are each 2-dimensional, they are not, unfortunately, in the same plane. To model fully the interaction between them would therefore require a 3-D model on the air side, interacting with the full surface of the 2-D membrane. While there are no conceptual difficulties with this, it was felt that the basic concept could be tested more economically, and sufficiently for present purposes, by assuming a 1-D membrane interacting with a 2-D acoustic “tube” (or rather “slit”, as the latter is assumed to be uniform – or infinitely long – in the third dimension).

The frequency response, Fig. 3, produced by the TLM model compares closely with the ana [4]. In both cases, average membrane displacemen nines the change in capacity between membrane a nteraction between the membrane and the short vo ane which the TLM system captures quite well. T LM and predicted response is that the resonant eff ly zero average displacement mode in the membrane, is less sharp in the numerical model, probably due to effects associ ated with the dynamic coupling of the two resonating systems and the boundary conditions at the microphone mouth.



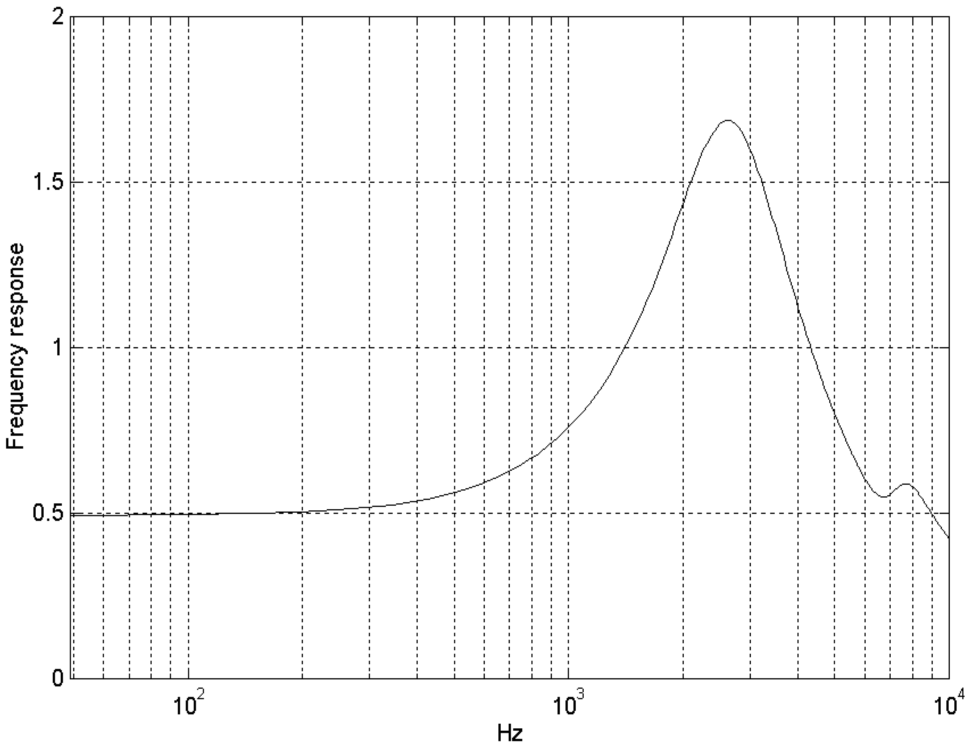


Fig. 3. Microphone frequency response (arbitrary units against frequency) from TLM simulation, with analytical result (insert) for comparison.

This idealised microphone model is somewhat removed from current commercial designs. For example, most modern microphones have a very small membrane-electrode gap, with a "squeeze-film" effect in the air between, which helps to ensure that flexing of the membrane is very small, at least up to fairly high acoustic frequencies. There will also be specially designed acoustic chambers and damping zones to try to achieve a desired response in both frequency and directional characteristics. The TLM modelling technique however can cope with these and other aspects of commercial designs, and work is under way to model more realistic configurations.

Computational loads are not high, particularly for small to medium sized devices: [5] gives some details. For example, all the present work was carried out on a mid-range personal computer.

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APPENDIX

Other areas of current TLM research at UCD

A. TLM modelling of the Euler-Bernoulli beam equation.

Assuming that the only restoring force is that due to bending, the small amplitude flexing of a beam gives rise to waves which obey the Euler-Bernoulli (or "biharmonic") equation, which is similar to the wave equation except that the spatial partial derivative is 4th order. The waves so described are dispersive: the wave speed increases with frequency. The flexural vibration of plates is governed by a two dimensional version of this equation, although with some additional complexity due to coupling of bending stiffnesses in two directions. Such flexural vibrations arise in modelling, for example, headphone diaphrams, loudspeakers cones and certain musical instrument components.

Can TLM be extended to model such vibrations? The author believes he has found a way to do so. During the coming months he hopes to implement the ideas in code to test them. The basic idea is again to stay close to the physical process, in which the flexing of the beam is interconnected with the transmission of tensional waves along the beam.

Such a novel method of modelling a vibratinvg beam is of interest in its own right. Furthermore, in the dynamic coupling of acoustic waves and flexural waves described above, the use of TLM to model both acoustic and flexural waves will be convenient.

Looking further, it seems likely that this new approach to modelling vibrating beams can be extended to more elaborate beam models, for example where shear forces also play a role, or where boundary conditions cause longitudinal as well as flexural waves to occur together.

B. TLM and fluid mechanics

Fluid mechanics involves the dynamic interaction of fluid pressure and velocity. Pressure distributions determine velocity flow patterns, and vice versa. CFD (computational fluid dynamics) is largely about modelling this dynamic interaction. While fluid particle velocities are frequently modest, changes in pressure on the other hand propagate at the speed of sound in the fluid.

The validity of the wave equation to describe this propagation of pressure waves in fluids depends on certain linearising assumptions known collectively as the "acoustic approximation". For example, one assumption is that bulk fluid velocities are negligible and the magnitude of the pressure variations are small in comparison with the background fluid pressure. The wave equation then emerges as a linearising approximation to the more general Eulerian equation of fluid flow:

TLM can be used to model the high speed pressure propagating effects, either by separating this "stiff" aspect of the problem from the low-speed, bulk flow aspects, or by modifying the basic TLM algorithm to take account of bulk flow effects. The second approach has been investigated in some detail. In summary, the idea is to add a pressure gradient to the TLM model, corresponding to the convective acceleration term in the Eulerian fluid model, thereby setting up a modified wave equation model.

C. Adding gradients and non-propagating DC components to TLM models

It would appear that the power, accuracy and flexibility of TLM can be significantly increased by allowing for the possibility of stationary (or quasi-stationary) potential distributions throughout the TLM mesh. The fluid pressure gradient mentioned above is one example of this idea. Another example is in modelling the telegrapher's equation, where it seems to provide a method of modelling a DC charge and/or current along the cable. Among other advantages, the new method allows the modelling of distributed (non-lumped) resistances, thereby avoiding the issues that arise when incorporating lumped resistances into the TLM scheme.

The fundamental TLM algorithm has to be reformulated somewhat, so that, for example, propagating pulses now represent *changes* in the main TLM variable. For lossless propagation, the main advantages is a physically more realistic model. But for lossy propagation and/or low frequency waves down to DC, whether electric, acoustic, mechanical or hydraulic, the accuracy, power and range of application of TLM seem to be significantly improved.